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Wednesday, May 09, 2012
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Z transform pairs:

Z - TRANSFORM PAIRS

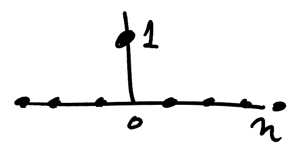
$x(n) = \delta(n) \longrightarrow 1 \quad \forall z$

$x(n) = \delta(n - n_0) \longrightarrow z^{-n_0} \quad z > 0$

$x(n) = a^n u(n) \longrightarrow \frac{1}{1 - az^{-1}}, \quad |z| > |a|$

$x(n) = -a^n u(-n-1) \longrightarrow \frac{1}{1 - az^{-1}}, \quad |z| < |a|$

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \qquad x[n] = \delta[n].$$



$$= 0z^{-2} + 0z^{-1} + 1z^{-0} + 0z^{1} + \cdots = 1$$

Z- trasform properties:

1.- linearity: $x[n] = x_1[n] + x_2[n]$

$$X(z) = X_1(z) + X_2(z)$$

2.- Convolution property

$$y[n] = x[n] * h[n]$$

$$Y(z) = X(z) \cdot H(z)$$

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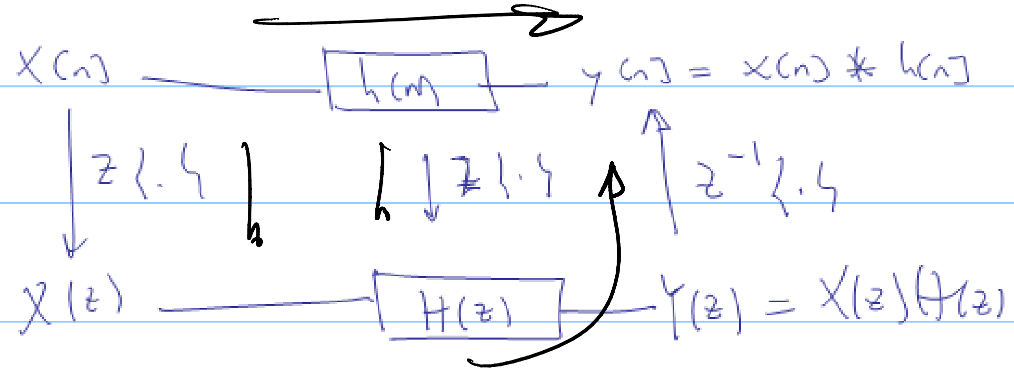
Indirect Convolution



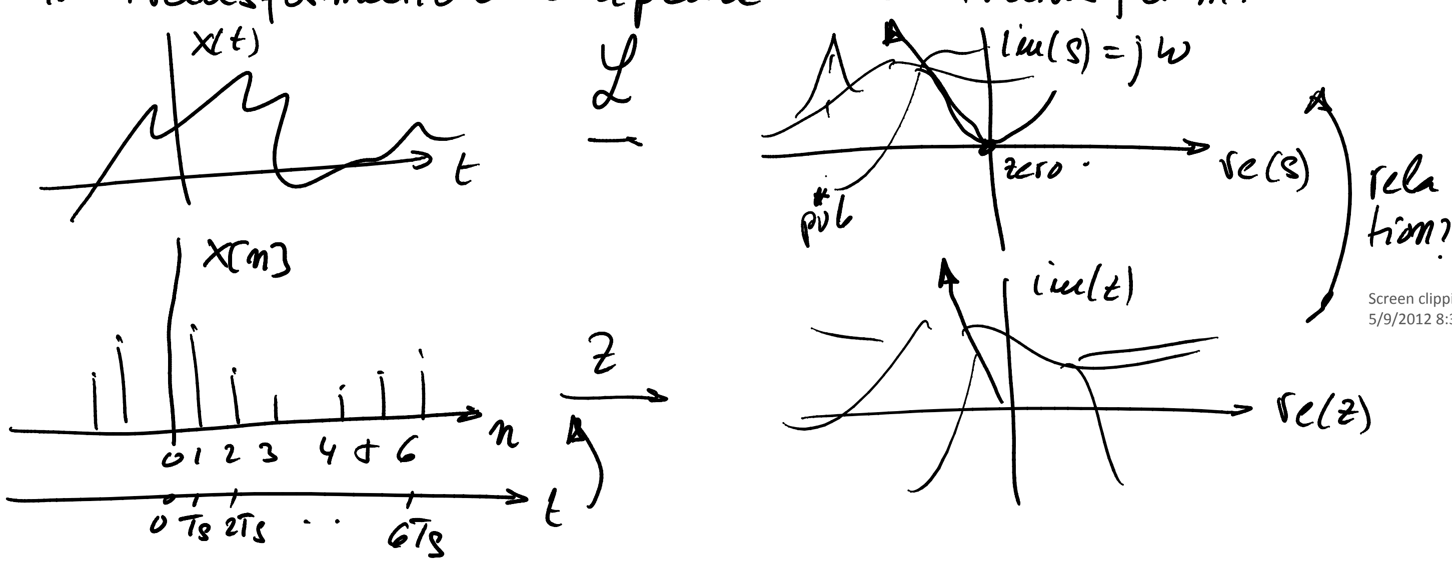
3.- Time shift property.

$$x[n] \longrightarrow X(z)$$

$$x[n-n_0] \longrightarrow X(z)\, z^{-n_0}$$

↳ phase shift.

4.- Transformation Laplace → Z-transform.



$$\mathcal{L}\{x_s(t)\} = \sum_{n=-\infty}^{\infty} x(nT_s)\, e^{-sT_s n}$$

$$z = e^{sT_s}$$

$$Z\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

$$z = e^{sT_s}$$

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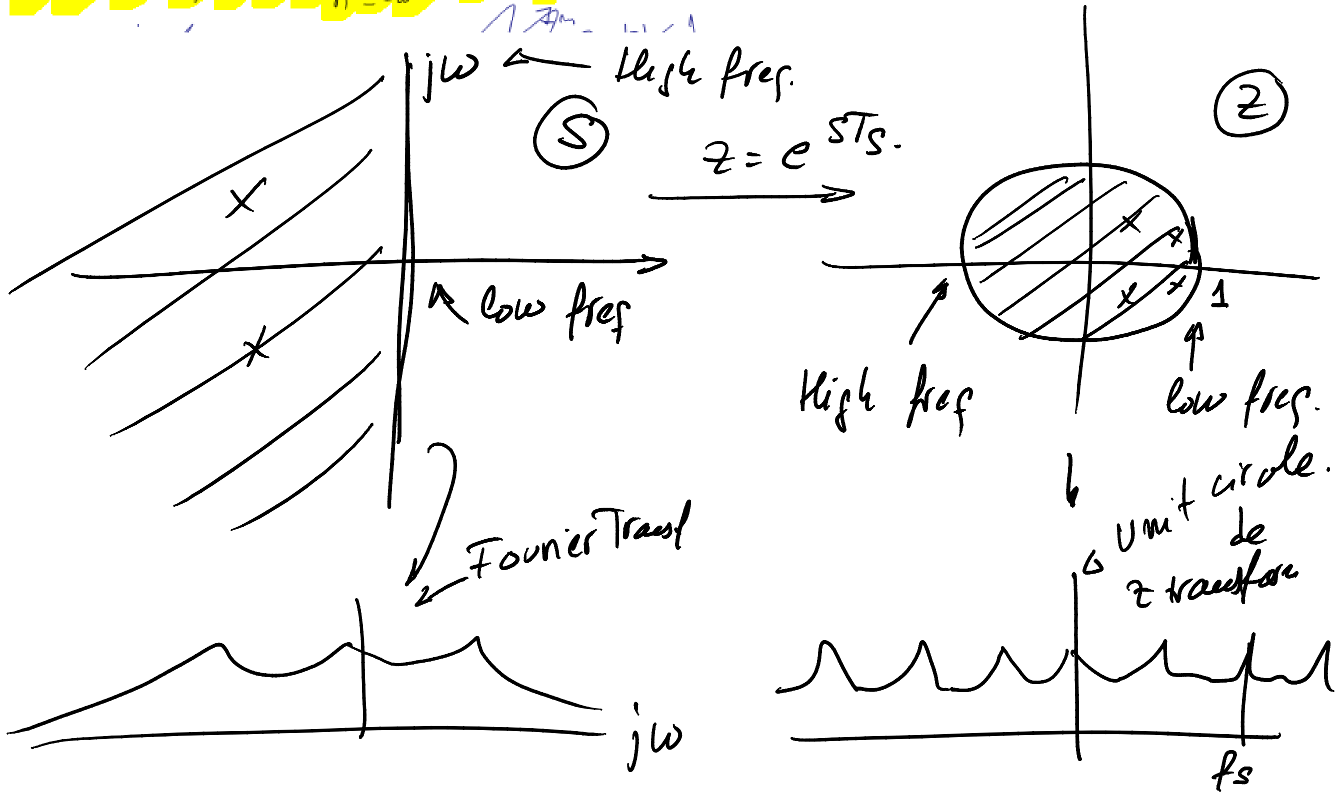
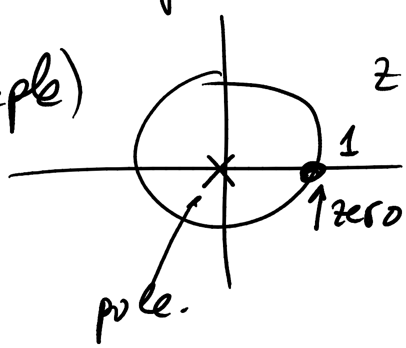
$Z\{x[n]\} = \sum_{n=-\infty} x[n] \ldots$



Relationship of z-transform, h[n] impulse resp, and block diagram.

- When designing a FIR filter (no feedback).

(1st example)



a) I want LPF.

$$H(z) = \frac{(z-1)}{(z-0)} = \frac{z-1}{z} =$$

$$= 1 - \frac{1}{z} = 1 - z^{-1}$$

b) $h[n] \rightarrow$ Inverse z-t of $H(z)$

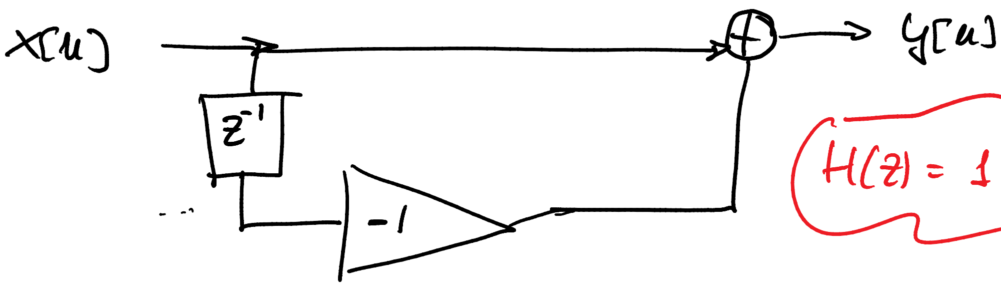
$h[n] = \delta[n] - \delta[n-1]$

c) block diagram.

c1) Obtain $y[n] = x[n] - x[n-1]$

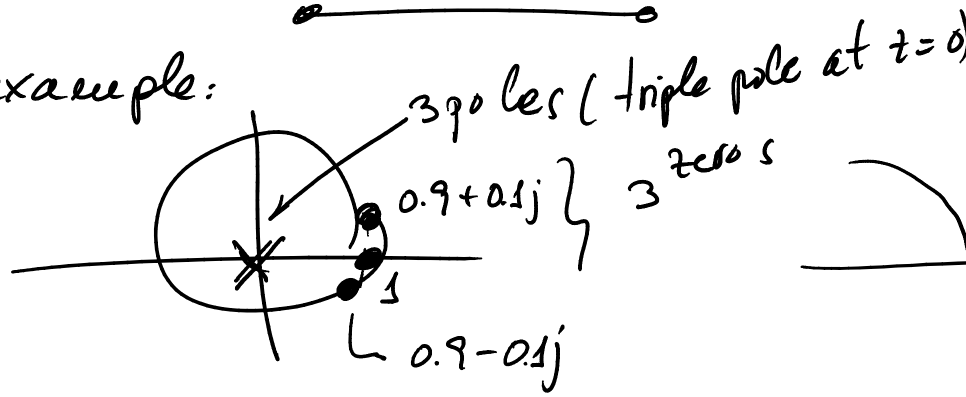
$$h[n] = \delta[n] - \delta[n-1]$$

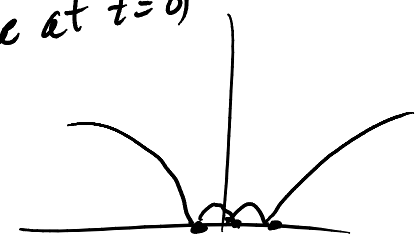
c) Block diagram:



$$H(z) = 1 - z^{-1}$$

Second example:





a) $$H(z) = \frac{(z-1)(z-0.9-0.1j)(z-0.9+0.1j)}{z^3} =$$

$$-0.1^2 j^2 = 0.1^2$$

$$= \frac{(z-1)\left(z^2 - 2\cdot 0.9z + 0.1jz - 0.1jz - 0.1j(0.1j) + -0.9\cdot 0.1j - 0.9(-0.1j)\right)}{z^3}$$

$$= \frac{(z-1)(z^2 - 1.8z + 0.1^2)}{z^3} =$$

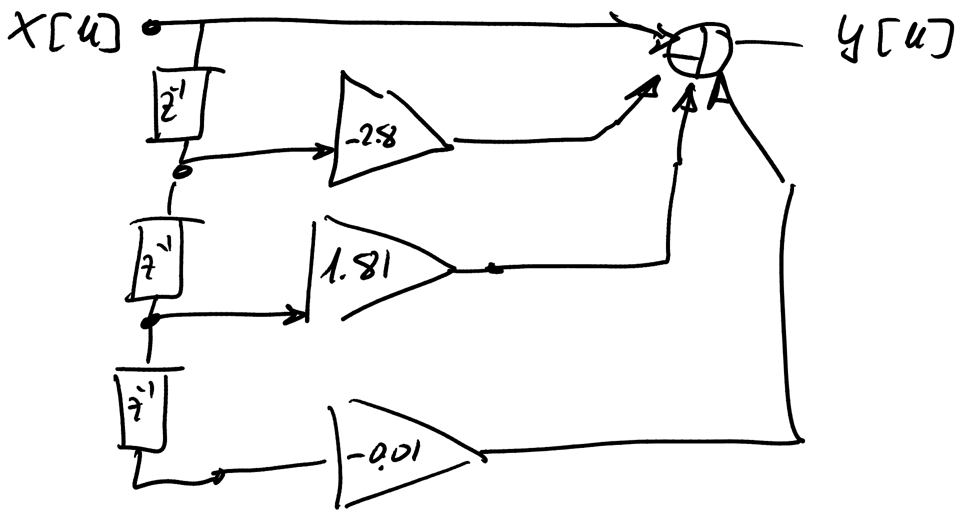
$$= \frac{z^3 - 1.8z^2 + 0.01z - z^2 + 1.8z - 0.01}{z^3}$$

$$= \frac{z^3 - 2.8z^2 + 1.81z - 0.01}{z^3}$$

$$H(z) = 1 - 2.8z^{-1} + 1.81z^{-2} - 0.01z^{-3}$$

2) $h[n] = \delta[n] - 2.8\,\delta[n-1] + 1.81\,\delta[n-2] - 0.01\,\delta[n-3]$

$y[n] = x[n] - 2.8\,x[n-1] + 1.81\,x[n-2] - 0.01\,x[n-3]$



Filters without feedback are not too good

Next day we will study IIR filters (feedback)