

Ztransform!!

Z - TRANSFORM

Definition $Z\{x[n]\} = \sum_{n=-\infty}^{+\infty} x[n] z^{-n} = X(z)$
 FOR CERTAIN R.O.C
 { For the values of z , SUCH THAT THE SUMMATION CONVERGES }
 R.O.C

Z is a complex variable, or $Z = \text{real}\{Z\} + j \cdot \text{imag}\{Z\}$

Example:

$x[n] = \left(\frac{1}{2}\right)^n u[n]$

$$X(z) = \sum_{n=-\infty}^{\infty} \left(\left(\frac{1}{2}\right)^n u[n]\right) z^{-n} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n 1 z^{-n} = \sum_{n=0}^{\infty} \left(\frac{z^{-1}}{2}\right)^n$$

$$= \frac{1}{1 - \frac{1}{2z}} \quad \text{iff } \left|\frac{1}{2z}\right| < 1 \rightarrow \frac{1}{|2z|} = \frac{1}{2|z|} < 1$$

\hookrightarrow R.O.C

$$= \frac{1}{1 - \frac{1}{2}z^{-1}} \quad \text{iff } \boxed{\frac{1}{2} < |z|}$$

$X(z) = \frac{z}{z - 1/2} \rightarrow$ zero $z=0$
 \rightarrow pole $z=1/2$

In general $x[n] = a^{-n} u[n] \quad a < 1.$
 $\forall r, \dots \quad z \quad \dots \quad \frac{1}{z} \quad \dots \quad \text{Roc } |a| < |z|$

$$X(z) = \frac{z}{z-a} = \frac{1}{1-az^{-1}} \quad \text{ROC } |a| < |z|$$

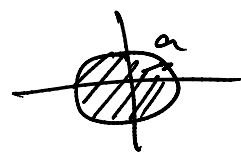
↳ pole $z=a$

$a^{-n} u[n]$ is causal is zero for $n < 0$.

$a^n u[-n-1] = X[n]$ ← anticausal.

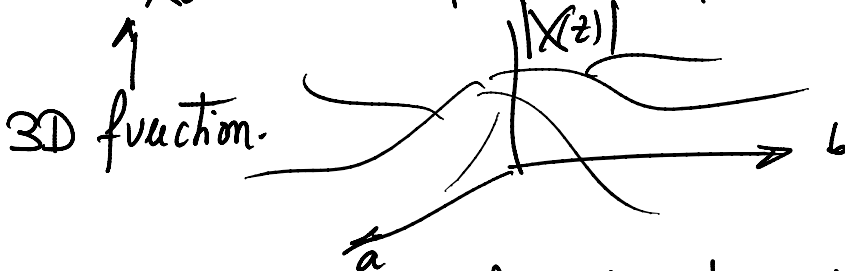
$$X(z) = \sum_{n=-\infty}^{\infty} (az^{-1})^n = \frac{1}{1-az^{-1}} \quad \text{ROC } |a| > |z|$$

↑
algebra



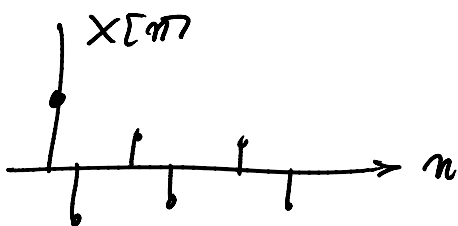
Z-plane.

$X(z)$ is a function of $z = a + jb$.

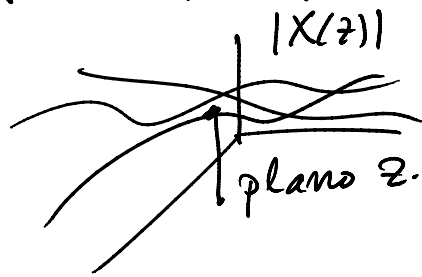


$X(z)$ may be complex.

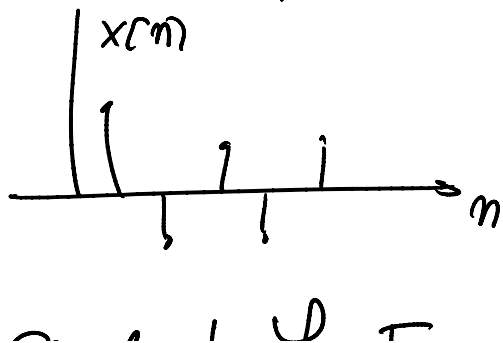
$$X(z) = |X(z)| e^{j\theta_X(z)}$$



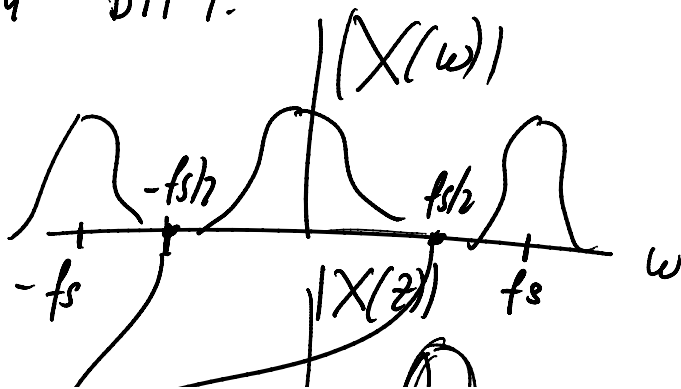
z^T



Relation of Z transform with DTFT.



DTFT



Similar to $\mathcal{L} \rightarrow F$

