Ee572039p1c2

Wednesday, April 04, 2012
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Todays Topics:

- Review of convolution from last class and cool trick.
- Review of DTFT.
  - Continuous Frequency property.
  - Replica of Spectrum due to the sampling in time.
  - Radians per sample instead radians per second, in the frequency domain.
- Review of Nyquist Sampling theorem.
- The DFT
  - Computational version of the DTFT.
  - Linear transformation of n points in time to n points in frequency.
  - Concept of W_n.
  - The F_n matrix and its application for the DFT.
  - Example of DFT using the F matrix.
  - The effect of zero padding.
  - Frequency resolution in the DFT.

Convolution trick:

$$x[n] = [1,2,3,4,5,5]$$
$$h[n] = [1,2,-1]$$

$$\begin{array}{rrrrrrrr}
 & & 1 & 2 & 3 & 4 & 5 & 5 \\
 & & & & & 1 & 2 & -1 \\
\hline
 & & -1 & -2 & -3 & -4 & -5 & -5 \\
 & 2 & 4 & 6 & 8 & 10 & 10 & \\
1 & 2 & 3 & 4 & 5 & 5 & & \\
\hline
1 & 4 & 6 & 8 & 10 & 11 & 5 & -5
\end{array}$$

$\underbrace{1 \quad 4 \quad 6 \quad 8}_{}$ Same as computed in last lecture.

With Matlab:

```
Command Window
>> conv([1 2 3 4 5 5],[1 2 -1])

ans =

     1     4     6     8    10    11     5    -5
```

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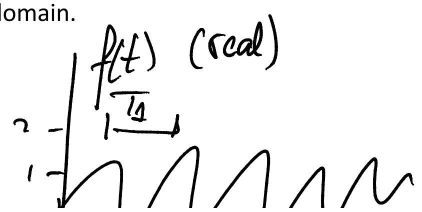
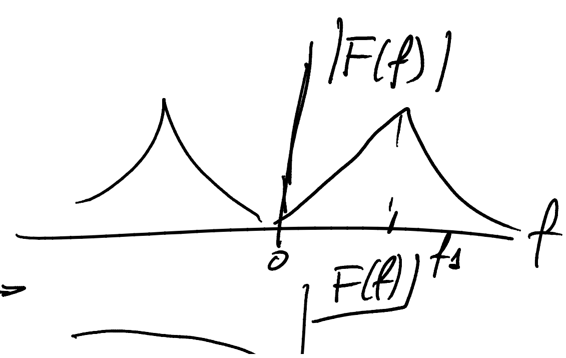
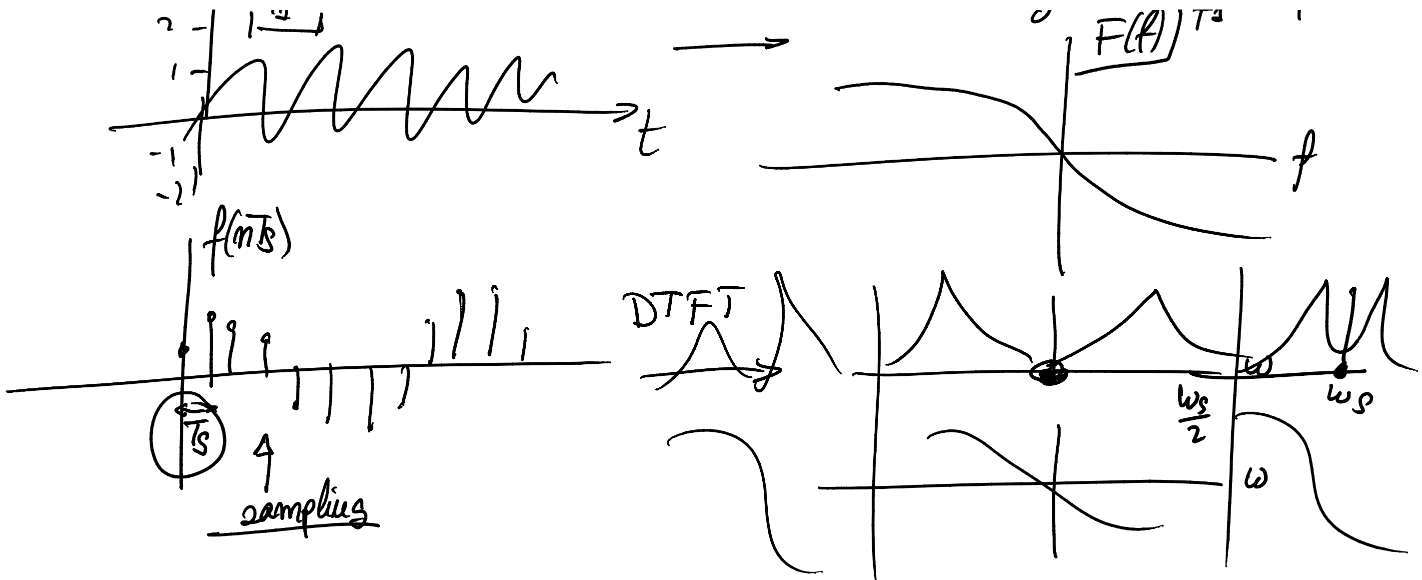
FT:



FT →





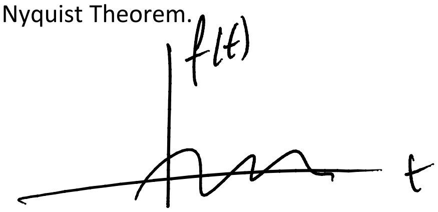
W_s sampling angular freq.

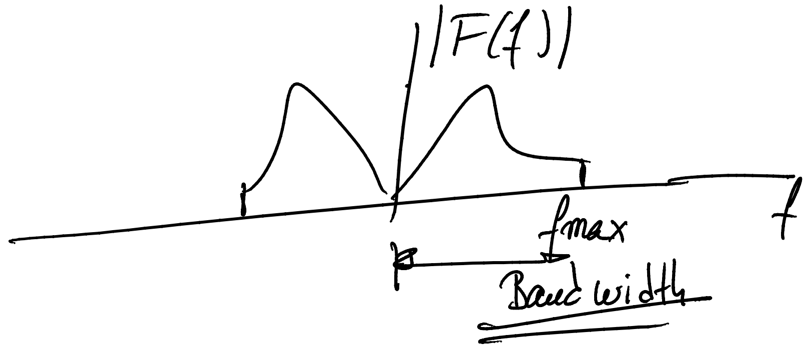
$W_s = \frac{2\pi}{T_s} = 2\pi f_s$ $\quad T_s \downarrow\downarrow \quad w_s \uparrow\uparrow$ (replicas separate)

We need $T_s \downarrow\downarrow$ such that there is not overlap

We want

Nyquist Theorem.

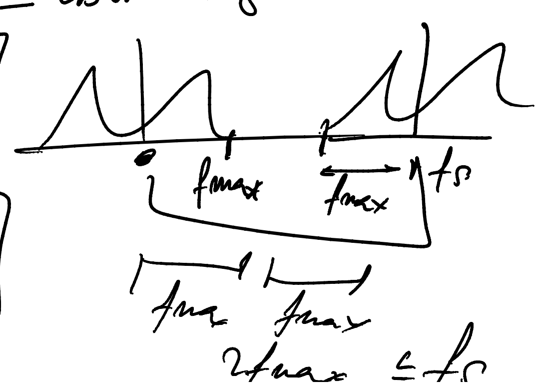




To be able to recover $f(t)$ from $f(nT_s)$ — discrete signal.

T_s must satisfy $\boxed{f_s \geq 2 f_{max}}$

$\boxed{\frac{1}{T_s} \geq 2 f_{max}}$



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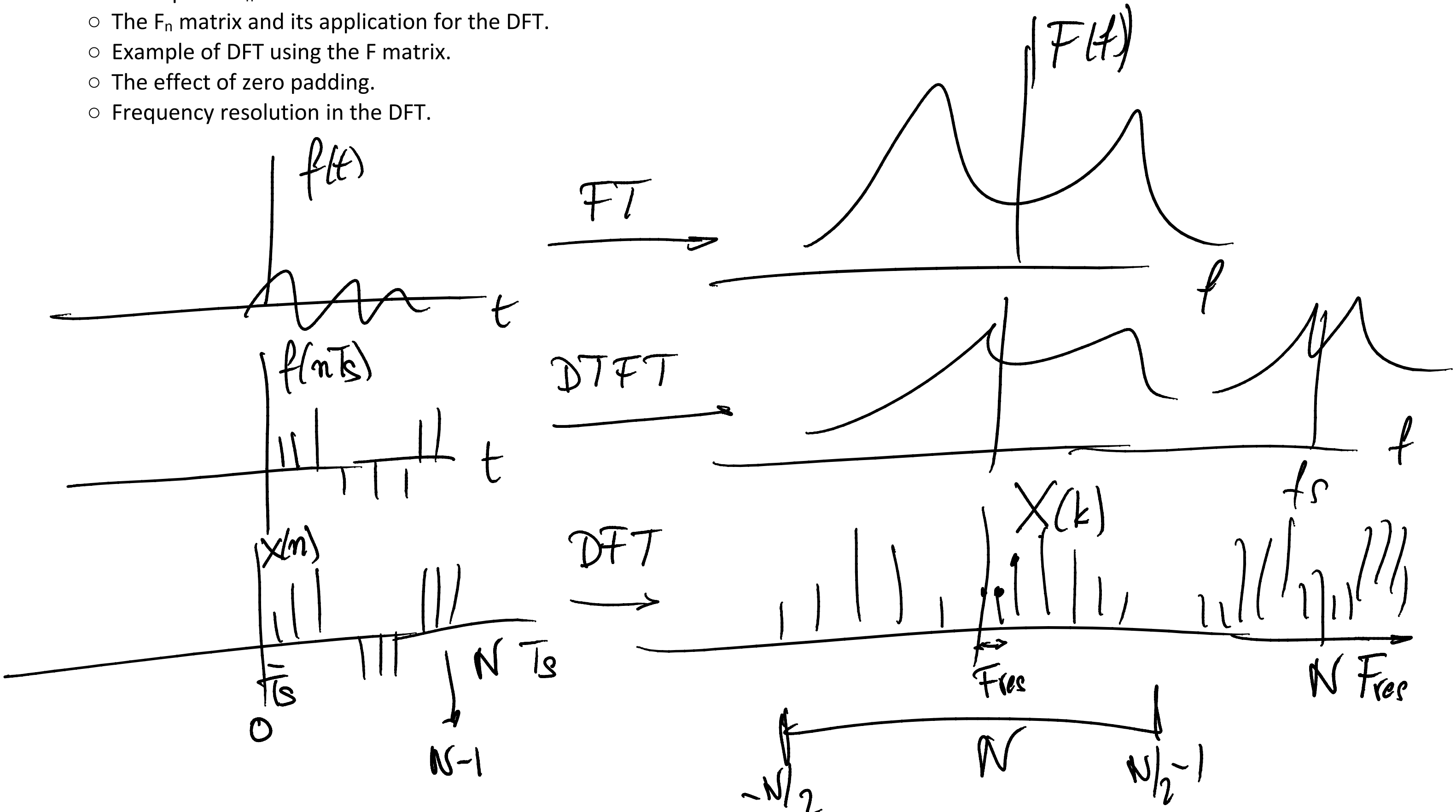

Linear Transf N points in time → N points in freq.

$$X(k) = \sum_{n=0}^{N-1} W_N^{kn} x[n]$$

$$\boxed{W_N^{kn} = e^{-j\frac{2\pi kn}{N}}}$$

N points

N points.

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} \\ F_{N \to 4} \\ \\ \end{bmatrix} \cdot \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ x[3] \end{bmatrix}$$

$\Big[$ 1,1 00 1,1 01 1,1 02 . . . . $\Big]$ DFT

$$F_N = \begin{bmatrix} W_N^{00} & W_N^{01} & W_N^{02} & \cdots & \\ W_N^{10} & & & & \\ W_N^{20} & & \ddots & & \\ \vdots & & & & W_N^{N-1\,N-1} \end{bmatrix}$$ <u>DFT Matrix</u>

$$F_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

<u>Example</u>

$$\begin{bmatrix} X \end{bmatrix} = \begin{bmatrix} & F_4 & \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

Frequency resolution in DFT.