EE 5720
Digital Signal Processing

Class p1c1:

Course Introduction.

- Chapter 1: Review of Digital Signals and Systems.

- **Profesor**: Dr. Luis M. Vicente.
- **Oficina:** DSP Lab. L 308B (PUPR).
- **Horas de oficina:** mediante cita.
- **Teléfono:** 787 622 8000 - ext: 344 **Fax:** 787 760 7815.
- **Correo electrónico :** lvicente@pupr.edu
- **Internet:** http://www.lmvicente.com/ee5720.htm

Libro de Texto

Introduction to Signal Processing (1996) S. Orfanides

Prentice Hall New York, N.Y ISBN: 0-13-209172-0

- **Proyecto 90%**
- **Asistencia: 10% (se pierde si faltan 3 o más veces)**

- **Proyectos:** el estudiante deberá realizar y presentar proyectos donde se apliquen los conceptos y técnicas aprendidas en el curso.
- **Asistencia virtual**: El estudiante deberá confirmar por e-mail cada semana que ha estudiado las notas y el vídeo antes de la siguiente clase para así poder seguir la dinámica de la clase. Para ello debe enviarme un e-mail antes de la siguiente clase. En caso contrario se apuntará una falta de asistencia. A las 3 faltas de asistencia el estudiante perderá un 10% de la nota final.

Reglas del curso

- Para enviar notificaciones necesito un e-mail con una dirección que incluya de algún modo su nombre y apellido (ej. boricua@coqui.com no es muy recomendable para localizar al estudiante en caso de emergencia).
- Los mensajes deberán titularse (subject) *ee5720 nombre apellido – (motivo del e-mail)*. Así evitaremos perdidas innecesarias.
- El estudiante debe estudiar las notas de clase y los videos de clase cada semana, deberá enviar un e-mail al profesor confirmándolo para que el profesor le apunte la asistencia virtual.
- Las fechas oficiales serán expuestas en el calendario del curso de la página web lmvicente.com/ee5720.htm.
- El estudiante es responsable de leer todos los anuncios expuestos en la página web y en BB, por favor activen su cuenta en BB Enterprise.

Objetivos y Temas:

Los estudiantes se familiarizaran con los siguientes temas:

- 1 Linear Shift Invariant Systems.
- 2. Discrete Linear Convolution.
- 3. Discrete Time Fourier Transform properties.
- 4. Sampling.
- 5. Discrete Fourier Transform. Properties
- 6. The FFT.
- 7. Cyclic convolution.
- 8. Introduction to Spectral Analysis.
- 9. Z-Transform. Properties.
- 10. Difference Equations.
- 11. Filter Structures.
- 12. Introduction to FIR filter design.
- 13. Introduction to IIR filter design.
- 14. Applications.

Class p1c1

- 1.1 Discrete Sequences.
- 1.2 Singularity Functions (Delta, Step, Decreasing Exp).
- 1.3 Review of Linearity.
- 1.4 Review of Time Invariance.
- 1.5 Signals as Linear Combination of Weighted Deltas.
- 1.6 LTI response to impulse (Delta).
- 1.7 Discrete Convolution.
- 1.8 FIR Discrete Filter Diagram.

1.1 Discrete Sequences

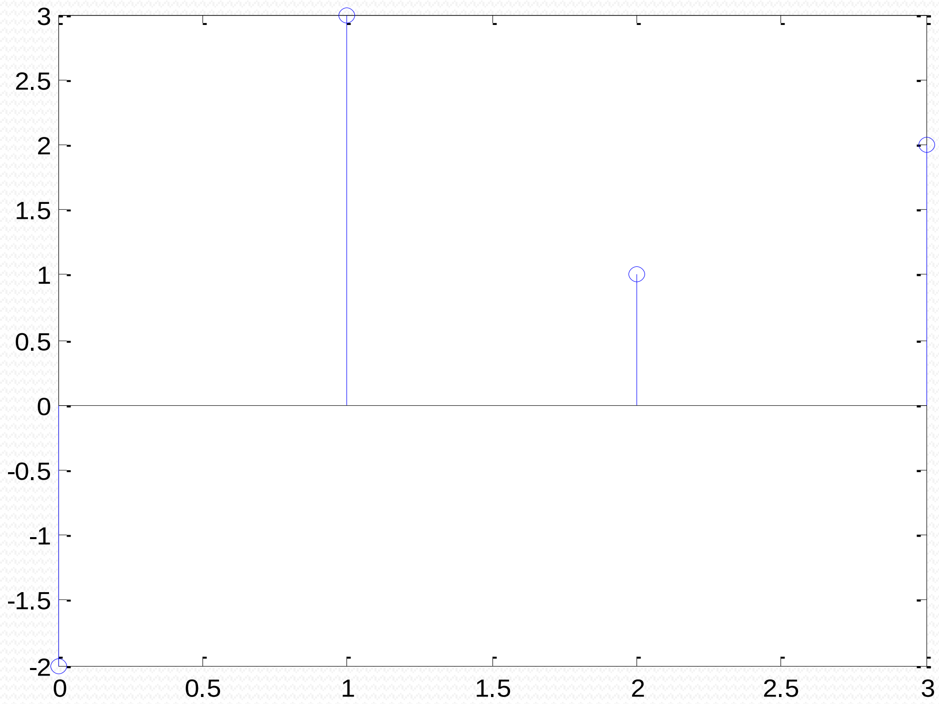
- Discrete sequences are the basic modeling of a discrete signal.
- A discrete signal is modeled as follows:
  - $x[n]=[x[0], x[1], x[2], x[3],\ldots.. x[k]]$
  - Here $x[n]$ is composed of $k+1$ samples.
  - Notice that the first sample is set at time $t=0$.
  - Each other samples are set at $t=nT$ where T is the sampling rate (for example $T=0.01$ second).
  - Since the sampling rate is known, we only keep the order of samples $n=0,1,2,\ldots.,k$. We assume the real time is nT.
  - *Example;* $x[n]=[-2, 3, 1, 2]$. Here we have four samples, from 0 to $n=3$, with values above exposed.
  - We could plot this signal using Matlab as shown below

1.1 Discrete Sequences

- *Example; x*[*n*]=[-2, 3, 1, 2] is a sequence where the first value we assume starts at n=0. n=[0 1 2 3].

Command Window

```
>> x=[-2, 3, 1, 2];
>> n=[ 0 1 2 3];
>> stem(n,x)
>>
```

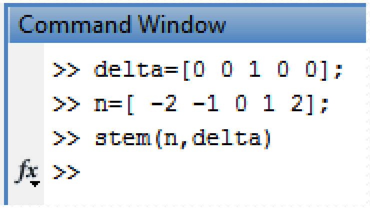


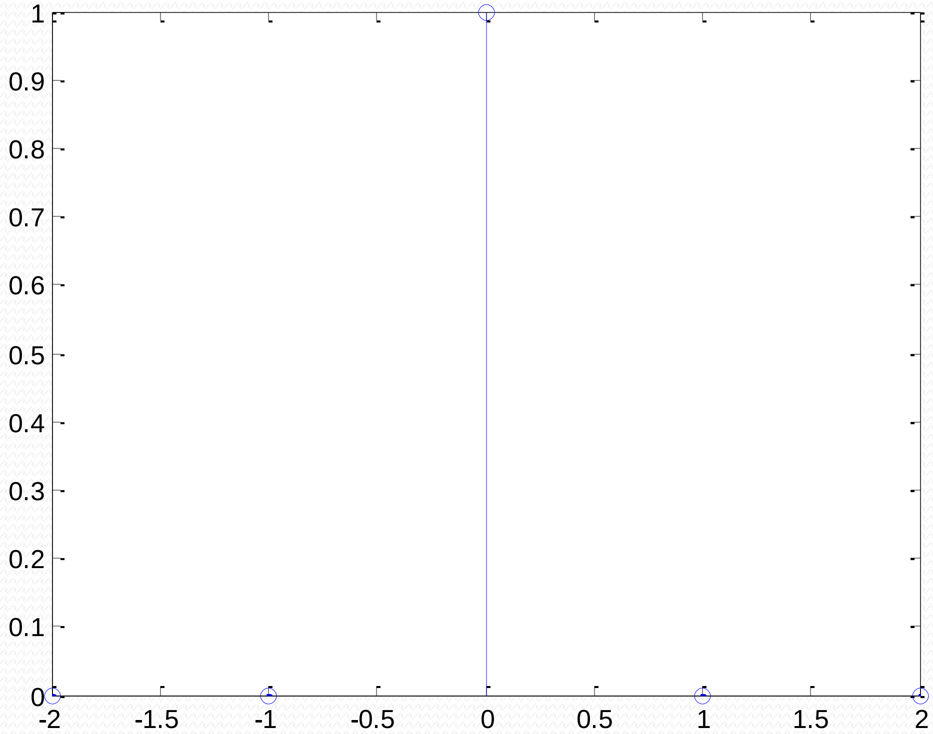
1.2 Singularity Funtions

- Discrete Delta (impulse) function.
  - Contrary to the analog delta function, the discrete delta function has amplitude of 1, instead of infinite amplitude for analog delta function.
  - The weighted discrete delta function has the amplitude of the weight!! For example $2\delta[n]$ has amplitude 2 at $[n]=0$ or $n=0$.
  - The shifted discrete delta $A\delta[n-25]$ has amplitude A at $[n-25]=0$ or $n=25$.
  - The shifted discrete delta $B\delta[n+35]$ has amplitude B at $[n+35]=0$ or $n=-35$.

1.2 Singularity Functions

- *Example; x*[*n*]=[0 0, 1, 0,0]. This is the delta function if the first sample starts at *n*=-2.



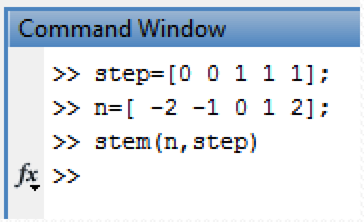


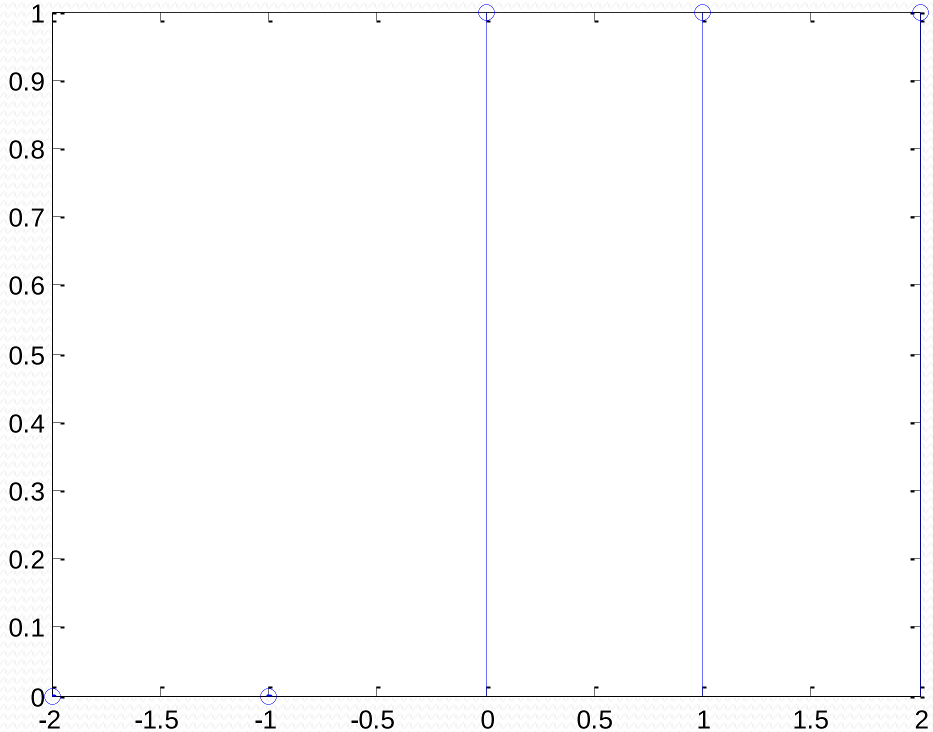
1.2 Singularity Funtions

- Discrete Unit Step function.
  - The unit step function has amplitude of 1, after sample 0.
  - The weighted unit step function has the amplitude of the weight!! For example 2*u*[*n*] has amplitude 2 at [*n*]>=0.
  - The shifted discrete unit step A*u*[*n*-25] has amplitude A at [*n*-25]>=0 or *n*>=25.
  - The shifted discrete delta B*u*[*n*+35] has amplitude B at [*n*+35]>=0 or *n*>=-35.

1.2 Singularity Functions

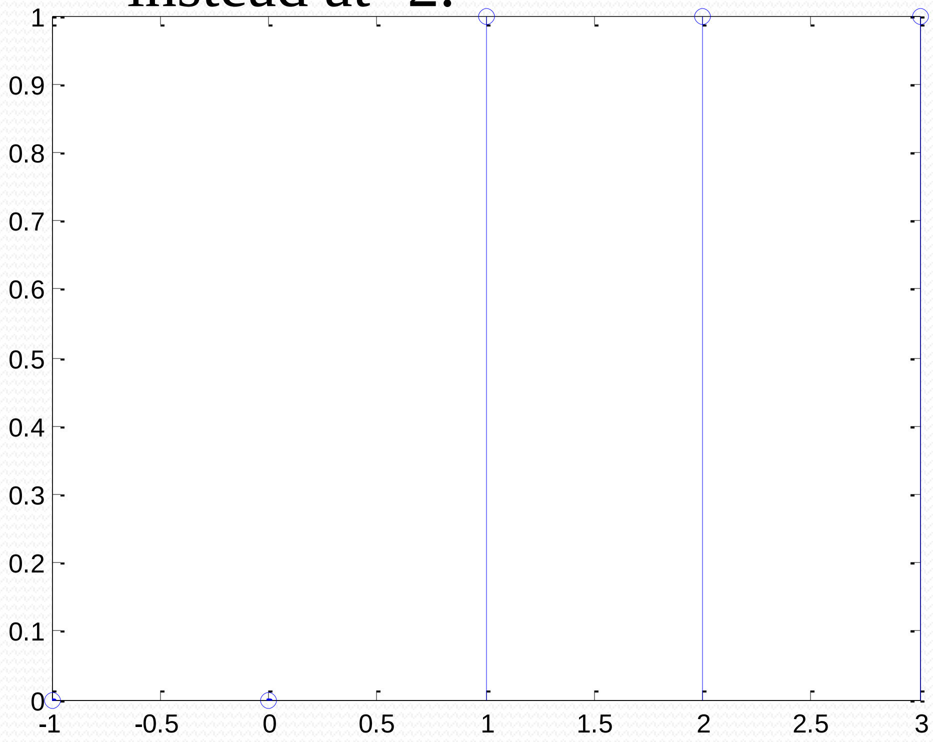
- *Example; u*[*n*]=[0, 0, 1, 1, 1]. This is the step function when first sample starts at -2.

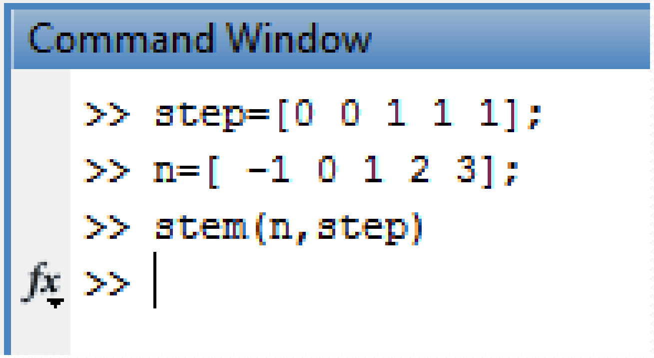




1.2 Singularity Functions

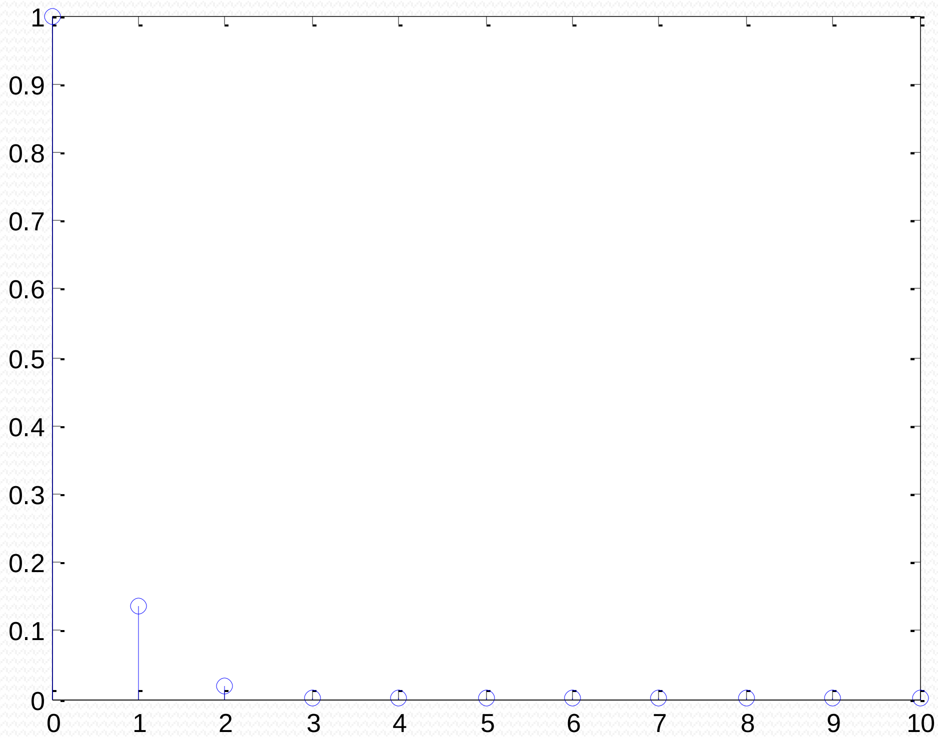
- To delay a signal means it starts later or earlier, therefore the first sample is different. For example, if the step in the previous example is delayed one sample the function is the same $u[n]$=[0, 0, 1, 1, 1]. But the first sample starts at -1 instead at -2.





1.2 Singularity Functions

- Decreasing exponential. Here we make use of an anonymous function defined as follows:



Command Window

```
>> decexp = @(n) exp(-2*n);
>> x = decexp(0:10);
>> stem([0:10],x)
>>
```

1.3 Review Linearity

- A linear system satisfy superposition and homogeneity.
- A system makes a transformation of an input signal, therefore the output signal is in general:

$$y[n] = T\{x[n]\}$$

- This means $x[n]$ is transformed in such a way that we obtain $y[n]$. Example: amplify the signal by 2. $T\{\}$ is the multiplier 2.

$$y[n] = 2x[n]$$

1.3 Review Linearity

- Superposition property of a system must satisfy that if:

$$y_1[n] = T\{x_1[n]\}$$

$$y_2[n] = T\{x_2[n]\}$$

- Then:

$$y_1[n] + y_2[n] = T\{x_1[n]\} + T\{x_2[n]\} = T\{x_1[n] + x_2[n]\}$$

- This means that applying a sum of signals to a system is the same to apply the system to each signal and them add the output of both systems.
- This allows us to decompose complicated signals in to known easy components and apply the system to each components.
- One decomposition is the Fourier Transform, where we decompose the signal in harmonics (pure sinusoids).

1.3 Review Linearity

- Homogeneity property of a system must satisfy that if:

$$y[n] = T\{x[n]\}$$

- Then:

$$ay[n] = aT\{x[n]\} = T\{ax[n]\}$$

- This means that applying the transformation to a signal multiplied by a constant *a* is the same that apply the system to the original signal and then add the multiplication factor at the output directly.

1.3 Review Time Invariance

- Time invariance says that the system does not change with time (there is not aging).
- Mathematically speaking it means that if we apply a system to a signal today,

$$y[n] = T\{x[n]\}$$

- and we apply the same signal later:

$$y[n-n_0] = T\{x[n-n_0]\}$$

- We obtain the same output (but at the *same later time* we applied the later signal). In this case later means a delay of n_0 samples.

1.5 Signals as Lin. Comb. of Weighted Deltas.

- Any signal could be written as a linear combination of deltas multiplied by particular factors. For example:

$$x[n] = [1, 3, 4, -2]$$

- Is the same as:

$$x[n] = 1\delta[n] + 3\delta[n-1] + 4\delta[n-2] - 2\delta[n-3]$$

- Since each delta is located at the exact place where the original amplitudes of the signal are located.
- This allows to use Linear Invariant Signals in a new way.

1.6 LTI response to impulse (Delta).

- A LTI system responds to a train of deltas as follows:

$$x[n] = 1\delta[n] + 3\delta[n-1] + 4\delta[n-2] - 2\delta[n-3]$$

$$y[n] = T\{x[n]\} = T\{1\delta[n] + 3\delta[n-1] + 4\delta[n-2] - 2\delta[n-3]\}$$

- Since the system is linear and time invariant, we apply the transformation to each delta. We call the response of a delta a special letter "*h*"

$$y[n] = 1T\{\delta[n]\} + 3T\{\delta[n-1]\} + 4T\{\delta[n-2]\} - 2T\{\delta[n-3]\}$$

$$y[n] = 1h[n] + 3h[n-1] + 4h[n-2] - 2h[n-3]$$

1.6 LTI response to impulse (Delta).

- This result is very important. It says we only need to obtain the response to a delta.
- Then decompose any signal as linear combination of deltas.
- The output is a linear combination of the response to a delta.

- **We just need to obtain $h[n]$ to be able to compute the output to ANY other signal!!!**

1.7 Discrete Convolution.

- Discrete convolution is nothing that above expression.
- Lets use a more general expression for an input signal decomposed in a train of deltas.

$$x[n] = x_0\delta[n] + x_1\delta[n-1] + x_2\delta[n-2] + x_3\delta[n-3] + ...$$

- This general expression could be written as:

$$x[n] = \sum_{k=0}^{\infty} x_k\delta[n-k]$$

- Of course, give several values to *k* and we end up with the original expression.

1.7 Discrete Convolution.

- When we apply a LTI system to this signal we have:

$$y[n] = T\{x[n]\} = T\left\{\sum_{k=0}^{\infty} x_k \delta[n-k]\right\}$$

- Since the system is Linear we could enter the T inside the summatory:

$$y[n] = T\{x[n]\} = \sum_{k=0}^{\infty} x_k T\{\delta[n-k]\}$$

- Because is Time Invariant, the transformation of a delta is the same at any delay, so:

$$y[n] = T\{x[n]\} = \sum_{k=0}^{\infty} x_k h[n-k]$$

1.7 Discrete Convolution.

- The last expression, repeated here, is called the linear discrete convolution:

$$y[n]=\sum_{k=0}^{\infty} x_k h[n-k]$$

- Also written as:

$$y[n]=\sum_{k=0}^{\infty} x[k]h[n-k]=\sum_{k=0}^{\infty} h[k]x[n-k]$$

- Convolution is as important as the sum or multiplication, so it has a particular symbol:

$$y[n]=x[n]*h[n]$$

1.7 Discrete Convolution.

- Example of convolution

$$x[n] = [1,2,3,4,5,5]$$

$$h[n] = [1,2,-1]$$

- The expression is:

$$y[n] = \sum_{k=0}^{\infty} x[k]h[n-k] = \sum_{k=0}^{\infty} h[k]x[n-k]$$

- The variable is *k* so we change our signal to variable *k* and we use the delta decomposition.

$$x[k] = 1\delta[k] + 2\delta[k-1] + 3\delta[k-2] + 4\delta[k-3] + 5\delta[k-4] + 5\delta[k-5]$$

$$h[k] = 1\delta[k] + 2\delta[k-1] - 1\delta[k-2]$$

1.7 Discrete Convolution.

- Now we are ready to perform convolution:
- Since $h[k]$ has only three terms, the summation will be only from k=0 to k=2.

- We start the convolution:

$$y[n] = \sum_{k=0}^{2} h[k]\,x[n-k]$$

1.7 Discrete Convolution.

- We start the convolution:

$$y[0]=\sum_{k=0}^{2} h[k]x[0-k]$$

- Recall:

$$x[k]=1\delta[k]+2\delta[k-1]+3\delta[k-2]+4\delta[k-3]+5\delta[k-4]+5\delta[k-5]$$
$$h[k]=1\delta[k]+2\delta[k-1]-1\delta[k-2]$$

- Then:

$$y[0]=\sum_{k=0}^{2} h[k]x[0-k]=h[0]x[0-0]+h[1]x[0-1]+h[2]x[0-2]$$

$$y[0]=1\bullet 1+2\bullet 0+-1\bullet 0=1$$

1.7 Discrete Convolution.

- We continue the convolution:

$$y[1] = \sum_{k=0}^{2} h[k]x[1-k]$$

- Recall:

$$x[k] = 1\delta[k] + 2\delta[k-1] + 3\delta[k-2] + 4\delta[k-3] + 5\delta[k-4] + 5\delta[k-5]$$
$$h[k] = 1\delta[k] + 2\delta[k-1] - 1\delta[k-2]$$

- Then:

$$y[1] = h[0]x[1-0] + h[1]x[1-1] + h[2]x[1-2]$$

$$y[0] = 1 \bullet 2 + 2 \bullet 1 + -1 \bullet 0 = 4$$

1.7 Discrete Convolution.

- We continue the convolution:

$$y[2]=\sum_{k=0}^{2} h[k]x[2-k]$$

- Recall:

$$x[k]=1\delta[k]+2\delta[k-1]+3\delta[k-2]+4\delta[k-3]+5\delta[k-4]+5\delta[k-5]$$
$$h[k]=1\delta[k]+2\delta[k-1]-1\delta[k-2]$$

- Then:

$$y[2]=h[0]x[2-0]+h[1]x[2-1]+h[2]x[2-2]$$

$$y[2]=1\bullet 3+2\bullet 2+-1\bullet 1=6$$

1.7 Discrete Convolution.

- We continue the convolution:

$$y[3]=\sum_{k=0}^{2} h[k]x[3-k]$$

- Recall:

$$x[k]=1\delta[k]+2\delta[k-1]+3\delta[k-2]+4\delta[k-3]+5\delta[k-4]+5\delta[k-5]$$
$$h[k]=1\delta[k]+2\delta[k-1]-1\delta[k-2]$$

- Then:

$$y[3]=h[0]x[3-0]+h[1]x[3-1]+h[2]x[3-2]$$
$$y[2]=1\bullet 4+2\bullet 3-1\bullet 2=8$$

1.8 FIR Discrete Filter Diagram.

- Diagram of a system is done with delay operators, gains and summs:

$$h[n] = [1, 2, -1]$$

Homework

- Read and understand chapter 1 of the book, completely.
- Try the chapter problems at the end of the chapter.
- Google your questions before addressing them to me, so we have a richer discussion and alternative answers.
- THANK YOU!!

Fin de la clase