Polytechnic University of Puerto Rico
Electrical Engineering Department

EE 4704-09: Analog Communication Systems

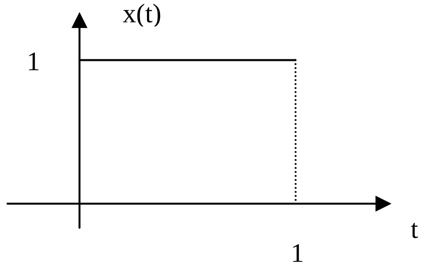
Quiz #1, Spring Quarter 2004, April 05

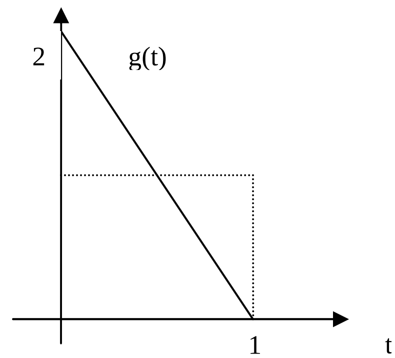
Name ____#______________

Write the results inside the boxes provided below. However, show all intermediate steps in a separate piece of paper. ***No credit will be given for final results only.***

Problem 1. *Topics: Signal Energy, Correlation Coefficient, Signal Component.(50 points)*

Given the following pulses $x(t)$ and $g(t)$ defined in the interval [0,1]:





a) *(10 points)*Find the mathematical expression of each signal.
$\boxed{x(t)=1 \ for\ t \in [0,1]}$ $\boxed{g(t)=-2t+2 \ for\ t \in [0,1]}$

b) *(10 points)*Find the energy in each pulse: $\boxed{E_g = \frac{4}{3}}$ $\boxed{E_x = 1}$

c) *(20 points)*Find the correlation coefficient: $\boxed{C_{xg} = \frac{\sqrt{3}}{2} = 0.866}$

d) *(10 points)*Find the "**component**" of $g(t)$ along $x(t)$ that holds the equation $g(t) \cong cx(t)$
$\boxed{c=1}$

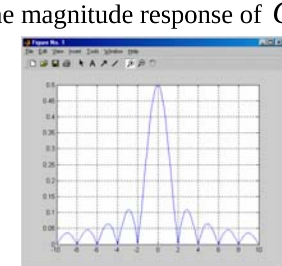
Problem 2. *Topics: Signal Bandwidth. (35 points)*

Lets be the signal $g(t) = rect(2t)$

a) *(10 points)*Find the Fourier Transform $G(f)$ of the signal $g(t)$ using any method you know.

Justify your answer. $\boxed{G(f) = \frac{1}{2}\frac{\sin(\pi \frac{f}{2})}{\pi \frac{f}{2}}}$

b) *(10 points)*Sketch approximately the magnitude response of $G(f)$.



c)

d) *(15 points)*Find the bandwidth of this signal if we consider all the Energy is contained by the first lobe. $\boxed{B = 2hz}$

Problem 3. *Topics: Signal Energy, Energy Spectral Density. (15 points)*

Given a communication system that we know is distortionless. Find the transfer function of the system if we know that the energy lost in the transmission is 25 and the delay response of the system is $12m\sec$.
NOTE: only correct reasoning and result of this problem will get full credit.

$$H(f) = \frac{1}{5} e^{j2\pi \frac{12}{1000} f}$$

Solution Problem 1.

a) The first signal is constant therefore $x(t) = 1$

The second signal is linear, therefore we just need two points to set the equation. For t=0 $g(t) = 2$ and for t=2 $g(t) = 0$ therefore the equation is $g(t) = -2t + 2$

b) The energy holds the formula $E_g = \int_{-\infty}^{\infty} g(t)^2 \, dt$ for real signals. Therefore

$$E_x = \int_0^1 1^2 \, dt = 1$$

$$E_g = \int_0^1 (-2t+2)^2 \, dt = \int_0^1 4t^2 - 8t + 4 \, dt = 4\frac{t^3}{3} - 8\frac{t^2}{2} + 4t \Big|_0^1 = \frac{4}{3} - 4 + 4 = \frac{4}{3}$$

c) $C_{xg} = \dfrac{\int_{-\infty}^{\infty} x(t) g^*(t) \, dt}{\sqrt{E_g E_x}}$

Let's find the inner product.

$$\int_{-\infty}^{\infty} x(t) g^*(t) \, dt = \int_0^1 1(2-2t) \, dt = 2 - t^2 \Big|_0^1 = 2 - 1 = 1$$

Therefore:

$$C_{xg} = \frac{1}{\sqrt{1\frac{4}{3}}} = \frac{\sqrt{3}}{2}$$

d) For the component the formula is

$$c = \frac{\int_{-\infty}^{\infty} x(t) g^*(t) \, dt}{E_x} = \frac{1}{1} = 1$$

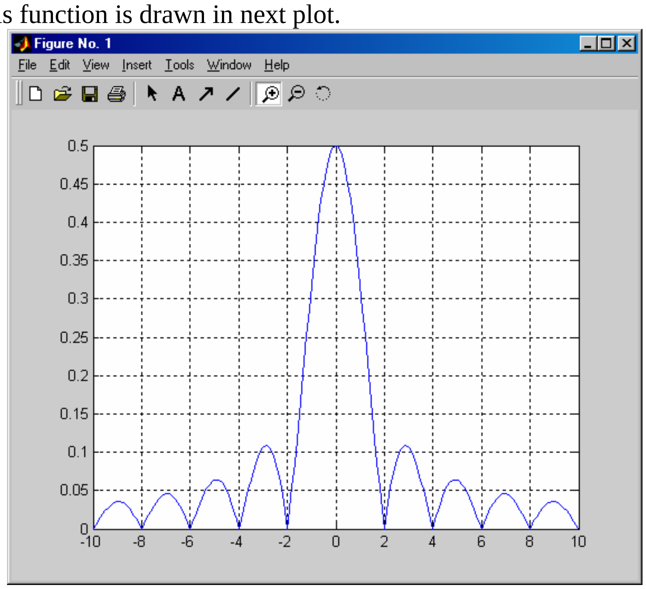
Solution Problem 2.

a) The Fourier transform of a $rect(t)$ is $\dfrac{\sin(\pi f)}{\pi f}$ if we look at the tables.

In our case we have $g(t) = rect(2t)$ which is a rectangle compressed. Because the Fourier Transform property of time scaling we have $x(at) \Leftrightarrow \dfrac{1}{|a|} X(\dfrac{f}{a})$

Where a=2. Therefore our Fourier Transform is $G(f) = \dfrac{1}{2} \dfrac{\sin(\pi \frac{f}{2})}{\pi \frac{f}{2}}$

b) The magnitude of this function is drawn in next plot.



c) The first lobe ends at $f = 2hz$. That is the Bandwidth

Solution Problem 3.

If the system is distortionless the signal after the system is just attenuated and delayed. If the input is $g(t)$, the output is $y(t) = Ag(t-t_d)$. The Fourier transform of this output signal is $Y(f) = AG(f)e^{+2j\pi t_d f}$

Because $H(f) = \dfrac{Y(f)}{G(f)}$ We have $H(f) = \dfrac{AG(f)e^{+2j\pi t_d f}}{G(f)}$

Therefore it has the form

$H(f) = Ae^{j2\pi t_d f}$ being $|H(f)| = A$

The system has a time delay of $t_d = 12m\sec$. Only A is left to calculate.

The problem states that the output energy is $E_y = \dfrac{1}{25}E_g$.

We know that the energy E_g is the integral of the ESD $\Psi_g(f)$, and the ESD is just

$\Psi_g(f) = |G(f)|^2$

The relation input output of ESD is

$\Psi_y(f) = |H(f)|^2 \Psi_g(f)$.

And from before we know that $|H(f)| = A$. Therefore

$\Psi_y(f) = A^2\Psi_g(f)$.

If we integrate we have

$$\int_{-\infty}^{\infty} \Psi_y(f)df = \int_{-\infty}^{\infty} A^2\Psi_g(f)df$$

$$\int_{-\infty}^{\infty} \Psi_y(f)df = A^2\int_{-\infty}^{\infty} \Psi_g(f)df$$

$E_y = A^2E_g$ Now with the equation $E_y = \dfrac{1}{25}E_g$ we find A

$\dfrac{1}{25}E_g = A^2E_g$ Therefore $A = \dfrac{1}{5}$

Solution $H(f) = \dfrac{1}{5}e^{j2\pi\frac{12}{1000}f}$