EE 1130

Freshman Eng. Design for Electrical and Computer Eng.

Class 4

Signal Processing Module (DSP).

- Laplace Transform. Transfer Function.
- Analog Filter Design with Zeros and Poles.
- Digital Filter Design with Zeros and Poles.

Laplace Transform.

- Working with Differential Equations (DE) is not easy. Laplace Transform allows exchange DE for something called Transfer Function (TF). The TF gives us a direct expression of Input/Output that the DE is not able to.

$$RC\dot{y} + y = x$$

- Also, it allows us to have an direct relation input/output!!

$$RC\frac{dy}{dt} + y(t) = x(t)$$

Laplace Transform.

- The Laplace Domain transforms time signals into vibration signals as follows:
  - In time: signal changes with time
  - In frequency: signal is view as its vibration/fequency components.

$$x(t) \rightarrow X(s)$$

$$y(t) \rightarrow Y(s)$$

$$\frac{dy}{dt} \rightarrow sY(s)$$

$$RCy(t) \rightarrow RCY(s)$$

Laplace Transform.

- Also, it allows us to have an direct relation input/output!!

$$RC\frac{dy}{dt}+y(t)=x(t)$$

- Aplying Laplace:

$$RCsY(s)+Y(s)=X(s)$$

- Now, there are only two variables $Y(s)$ and $X(s)$

Laplace Transform.

- Operating, we obtain a Direct Input/Output relationship

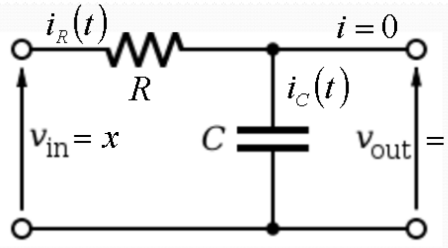
$$RCsY(s)+Y(s)=X(s)$$

$$Y(s)\left(RCs+1\right)=X(s)$$

$$Y(s)=\frac{1}{RCs+1}X(s)$$

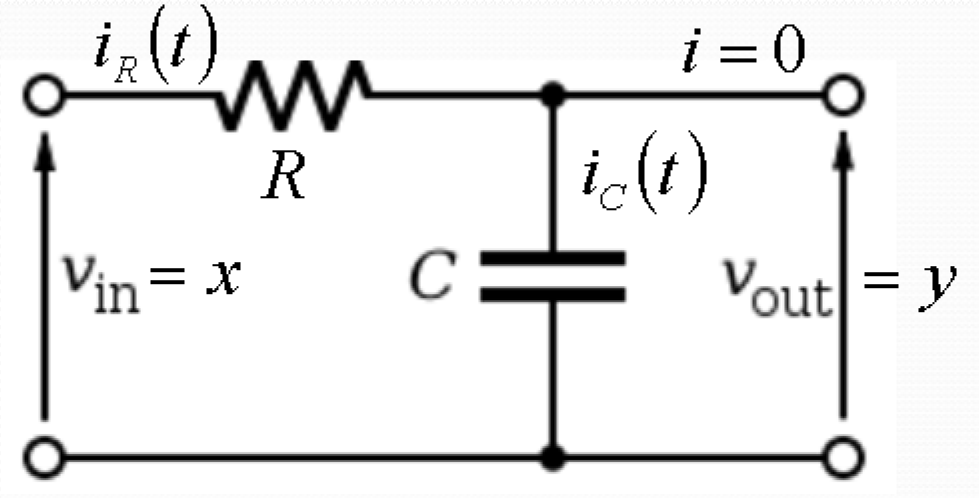
- We could easily implement this in Simulink!!!
- The multiplier of $X(s)$ is called Transfer Function.

$$H(s)=\frac{1}{RCs+1}$$



Laplace Analisys of Circuits.

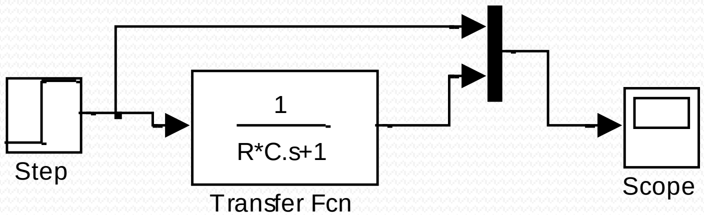
- Using Laplace we could find the Transfer function at once!!

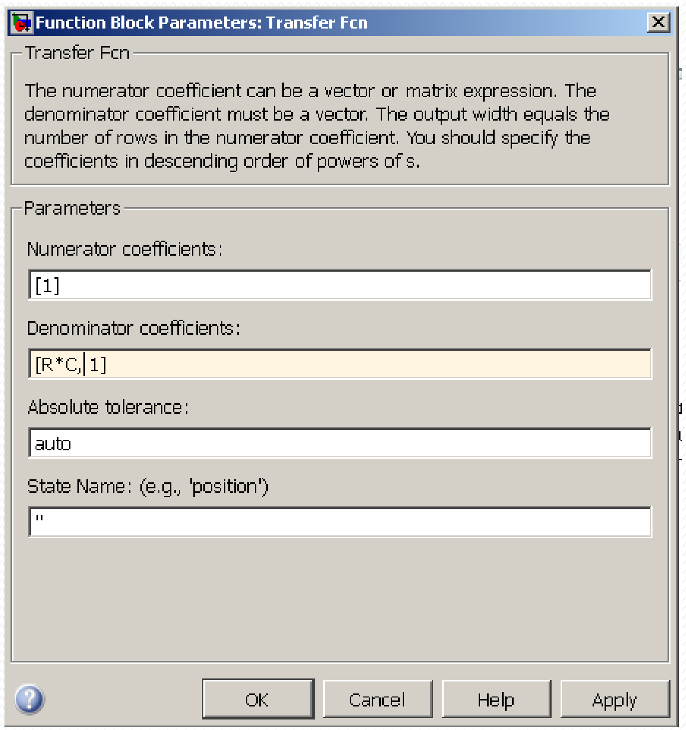


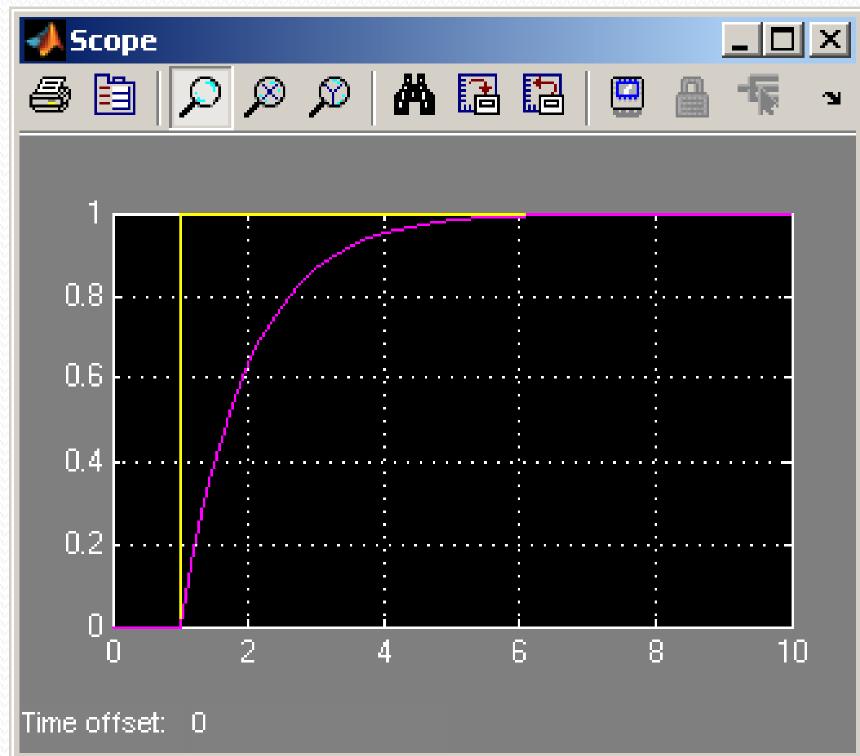
Simulink: Laplace Transform.

- Double click on Transfer Fcn to open options as shown below:
- Simulating:

$$H(s) = \frac{1}{RCs + 1}$$

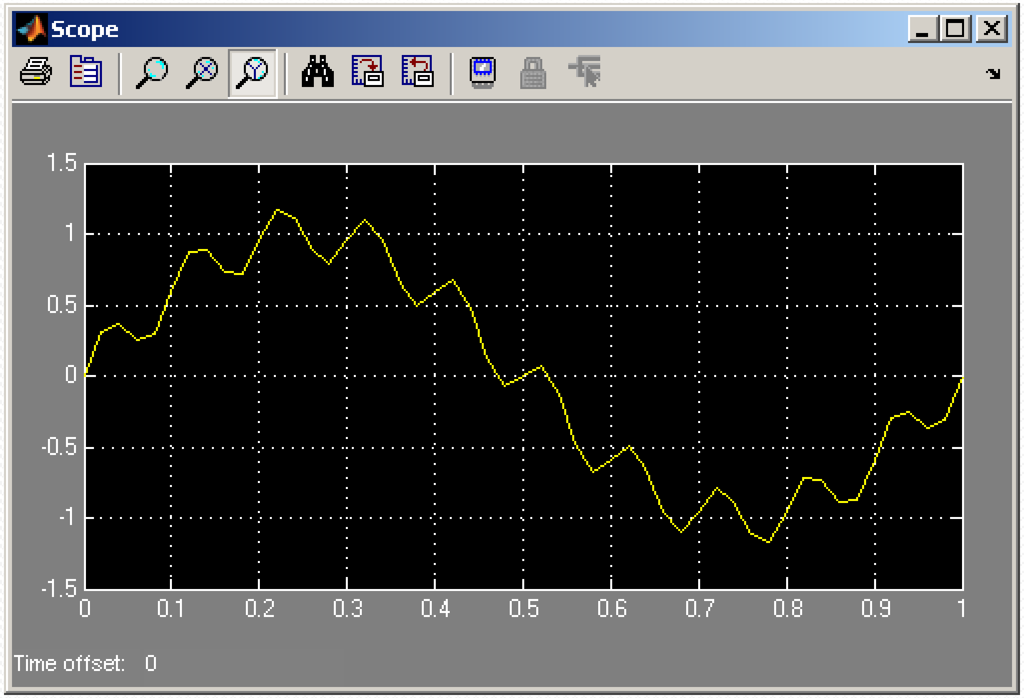






Simulink: Signal Processing.

- Last lecture we ended up with a noisy signal as next figure shows:



$$x(t) = \sin(2\pi 1t) + 0.2\sin(2\pi 60t)$$

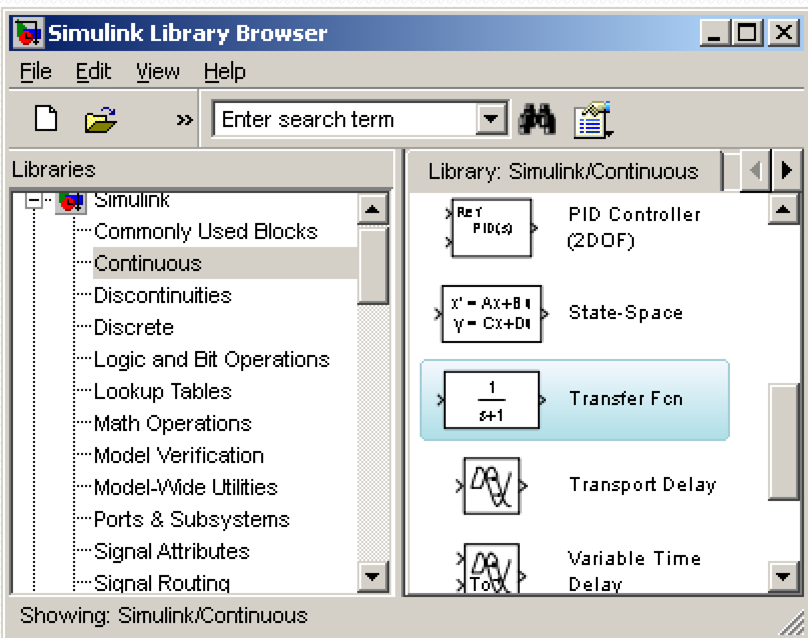
Design of Analog Filter.

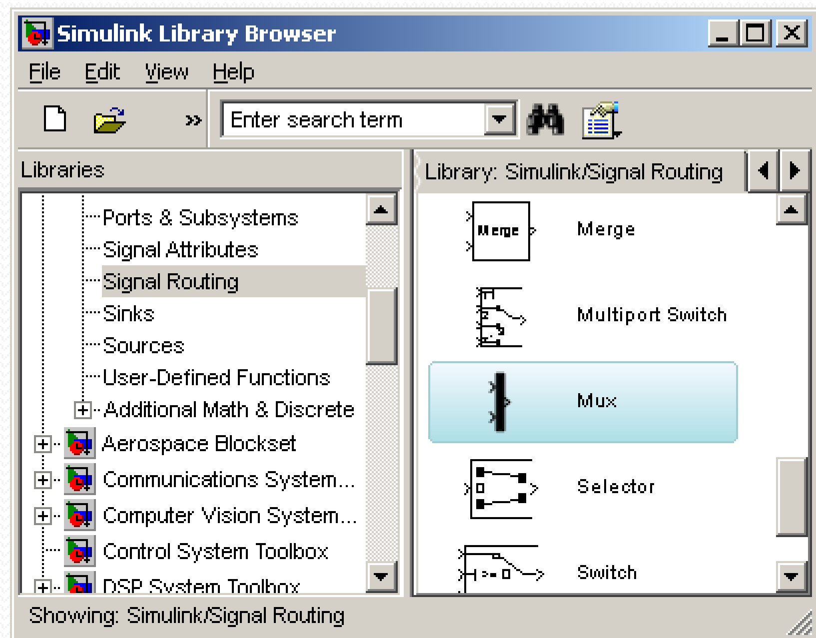
- Now we have two different tools to analyze an Electrical System (Electrical Filter, Electrical Circuit)
  - Differential Equation (time domain).
  - Transfer Function (vibration/frequency domain).

- When an Engineer needs to design an Electrical System to perform a particular task, the process is the inverse to analysis.
- This process is called Synthesis.
- When designing an analog filter:
  - WE START WITH THE TRANSFER FUNCTION and we end with an Electrical Circuit.

Simulink: Signal Processing.

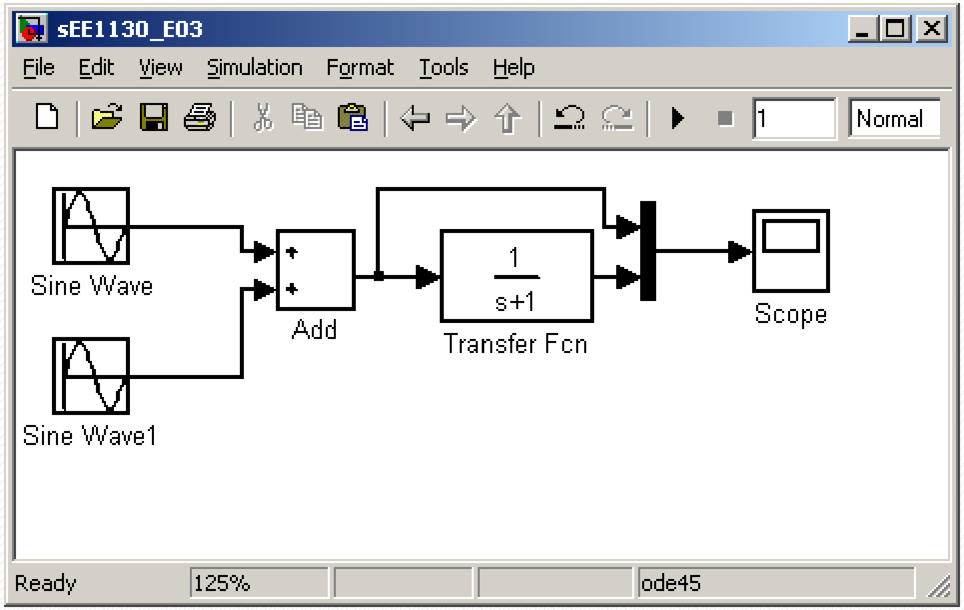
- We will insert a system that will filter out the ripple.
- First option is to insert from the continuous library group a Transfer Function block.
- We also add a Mux from Signal Routing library group.





Simulink: Signal Processing.

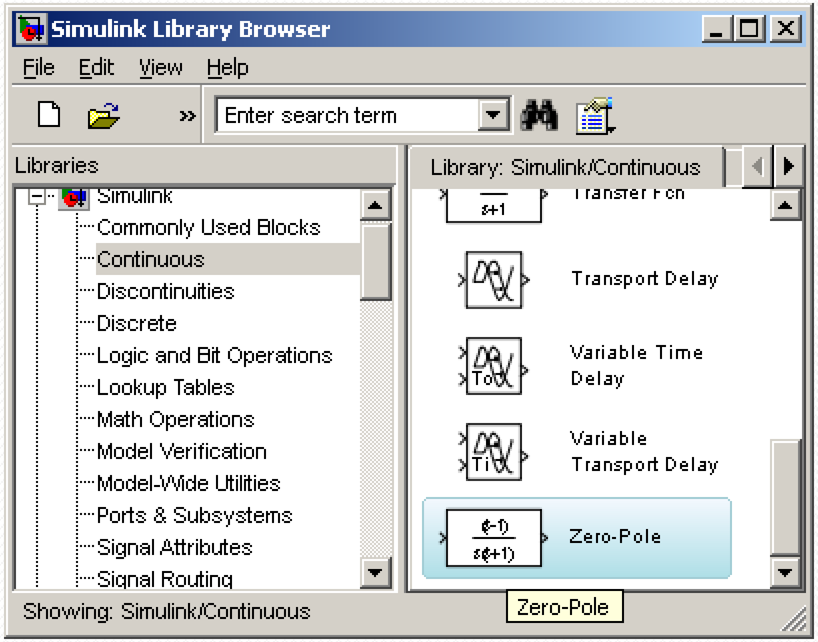
- We insert the Transfer Function after the summator and before the Mux.
- The Mux will allow the Scope to show two traces:

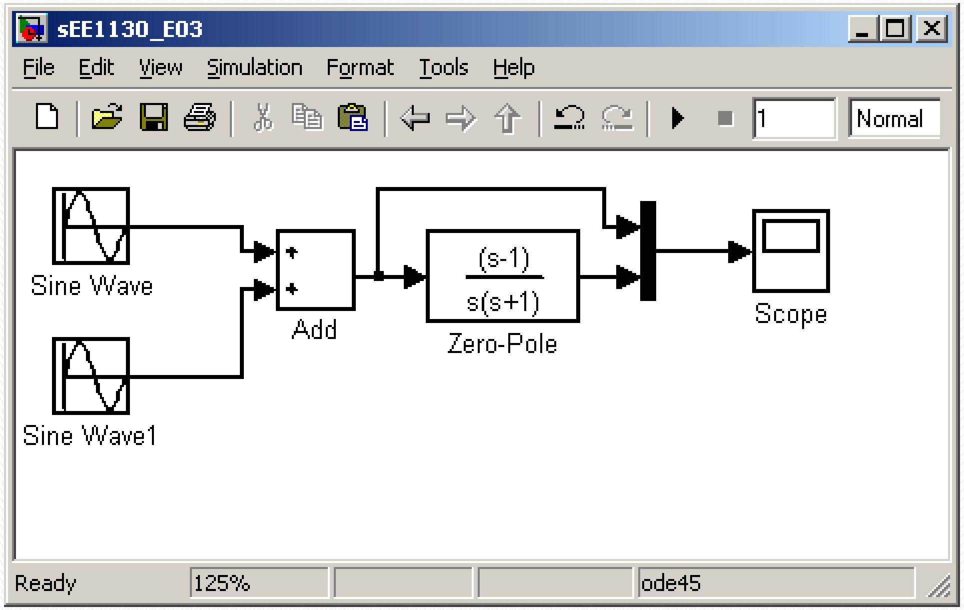


- Now, hit play and see:

Zero Pole Filter Design

- Now, lets design a filter that particularly eliminates the signal of 60Hz and keep untouched the signal of 1Hz. We do that using the Zero-Pole Transfer function





Zero Pole Filter Design

- **When you will study Filter Theory** you will learn that the roots of the numerator (**called zeros**) must be $s=2\pi 60j$ where 60 is the frequency to eliminate at the output.

Zero Pole Filter Design

- **When you will study Filter Theory** you will learn that one of the roots of the numerator (**called zeros**) must be $s_{z1}=2\pi 60j$ where 60 is the frequency to eliminate at the output.

$$Y(s)=\frac{(s-s_{z1})(s-s_{z2})\dots}{den}X(s)$$

- Observation: if you wanted also to kill a frequency of 100HZ you must set another zero/root to be $s_{z2}=2\pi 100j$
- Another problem is to set the denominator coefficients. And it is more dangerous. Because if you set some values of s to make the denominator zero, we explode the system.

Zero Pole Filter Design

- What are the values I must set at the denominator.
- I cannot set the denominator roots to the frequencies I want to amplify or let go untouched, because the system will go unstable.

$$Y(s) = \frac{(s - 2\pi 60 j)}{(s - 2\pi 1 j)} X(s)$$

- What I do is set the real part to avoid that singularity. For example set it to 300:

$$Y(s) = \frac{(s - 2\pi 60 j)}{(s + 300 - 2\pi 1 j)} X(s)$$

Zero Pole Filter Design

- To set the real part properly you will need to learn more about analog filter design. We do not have time in this class to discuss.

$$Y(s) = \frac{(s - 2\pi 60 j)}{(s + 300 - 2\pi 1 j)} X(s)$$

- Lets test it:

$$Y(s) = \frac{(2\pi 60 j - 2\pi 60 j)}{(2\pi 60 j + 300 - 2\pi 1 j)} X(2\pi 60 j) = \frac{0}{(2\pi 60 j + 300 - 2\pi 1 j)} X(2\pi 60 j) = 0$$

$$Y(s) = \frac{(2\pi 1 j - 2\pi 60 j)}{(2\pi 1 j + 300 - 2\pi 1 j)} X(2\pi 1 j) = \frac{377 j}{(377 j + 300)} X(2\pi 1 j) \cong \frac{1}{2} X(2\pi 1 j)$$

Zero Pole Filter Design

- But the coefficients of the numerator are some of the values of the Electrical Components.
- Remember, for the RC circuit we had:

$$Y(s) = \frac{1}{\tau s + 1} X(s) \qquad \tau = RC$$

$$H(s) = \frac{1}{RCs + 1}$$

$$Y(s)s + \frac{1}{RC} Y(s) = X(s)$$

$$\frac{dy}{dt} + \frac{1}{RC} y(t) = x(t)$$

Zero Pole Filter Design

- But the coefficients of the numerator are some of the values of the Electrical Components or amplifier gains.
- However, we can not have imaginary coefficients, because they are component values or amplifier gains that MUST BE REAL.
- We need to do a mathematical trick to convert imaginary numbers into real numbers!!
  - COMPLEX CONJUGATE
  - $(a + jb)\ (a - jb) = a^2 + b^2$ eso es debido a que $-j^2 = 1$

Zero Pole Filter Design

- When studying Filter Theory you will learn that the roots of the numerator must be (s-$2\pi 60j$) and (s+$2\pi 60j$). The use of complex conjugated roots is to have real coefficients because:

$$(s-2\pi 60j)(s+2\pi 60j)=s^2+4\pi^2 60^2$$

- At the denominator we just set roots (**poles**) to.

$$(s+340)(s+360)$$

- If you set smaller roots, the output becomes too large. Please try other values to check out by yourself

Zero Pole Filter Design

- The final transfer function is.

$$H(s)=\frac{s^2+4\pi^2 60^2}{(s+340)(s+360)}=\frac{s^2+142{,}120}{(s+340)(s+360)}$$

- Lets test it:

$$Y(s)=\frac{(2\pi 60j)^2+142{,}120}{((2\pi 60j)+340)((2\pi 60j)+360)}X(2\pi 60j)=\frac{0}{((2\pi 60j)+340)((2\pi 60j)+360)}X(2\pi 60j)=0$$

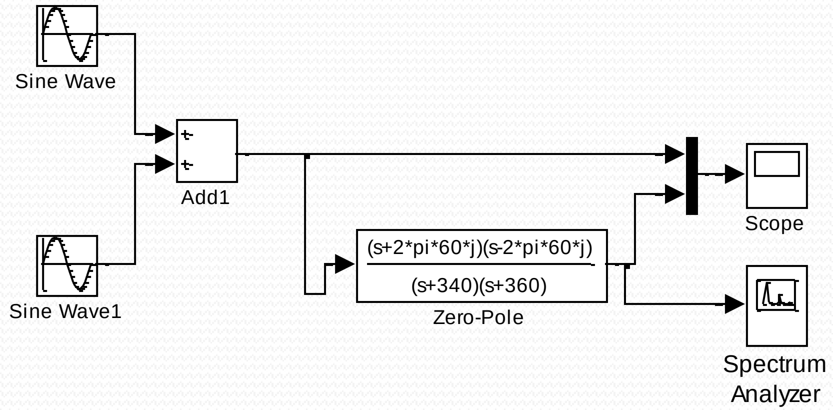
$$Y(s)\cong\frac{142{,}120}{(340)(360)}X(2\pi 1j)=1.1*X(2\pi 1j)$$

Zero Pole Filter Design

- The final Transfer Function that solve our problem is:

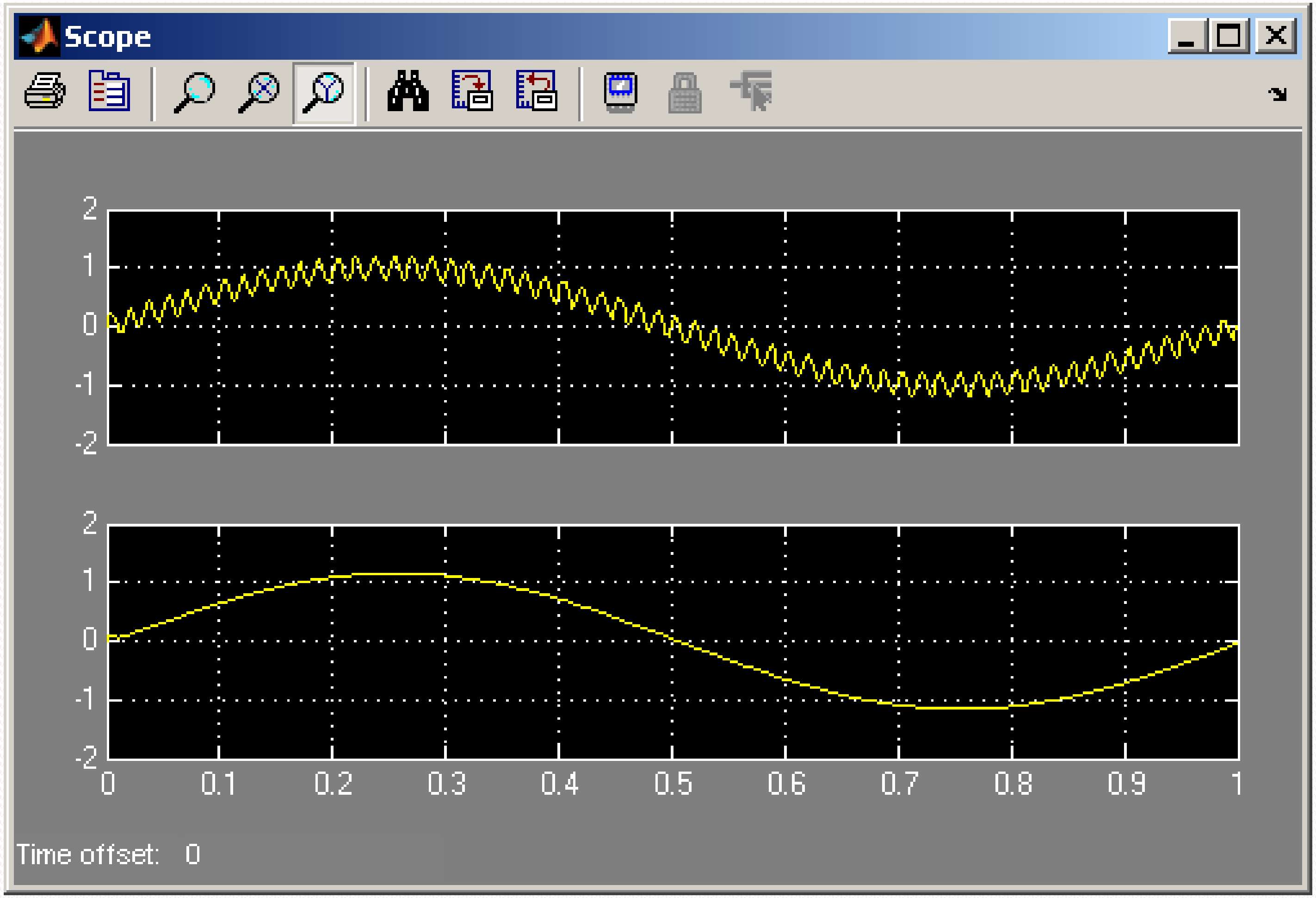
$$H(s) = \frac{s^2 + 142120}{(s+340)(s+360)}$$

- Now, we simulate this in Simulink



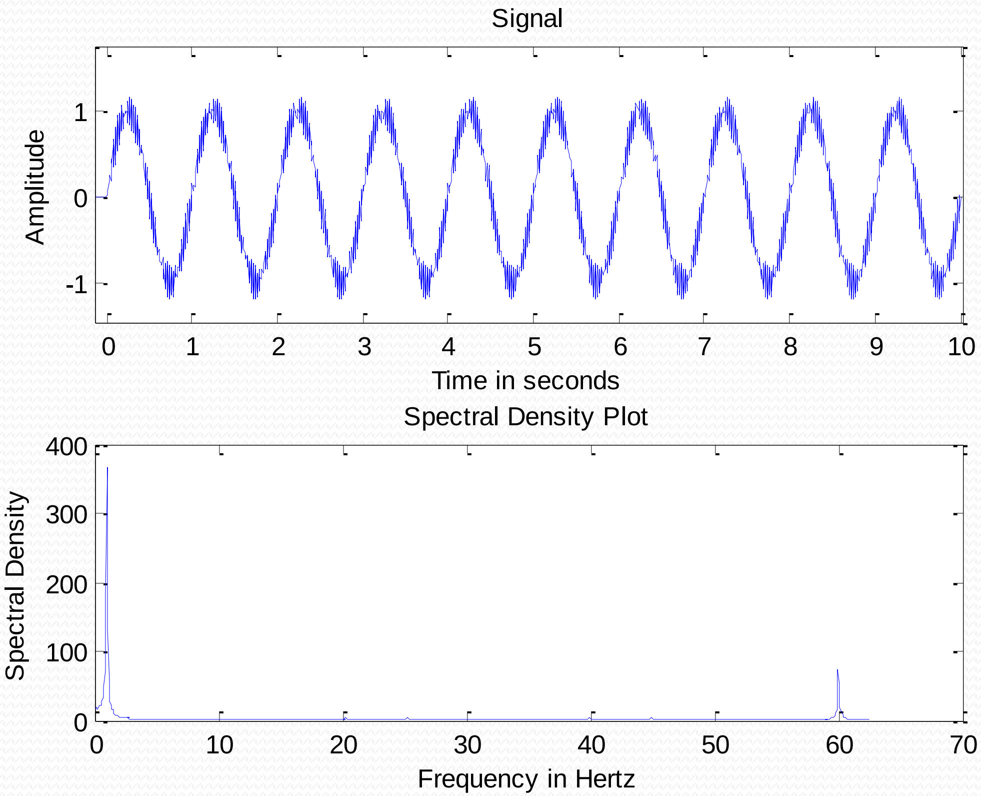
Zero Pole Filter Design

- Now we hit play and compare input and output in the Scope



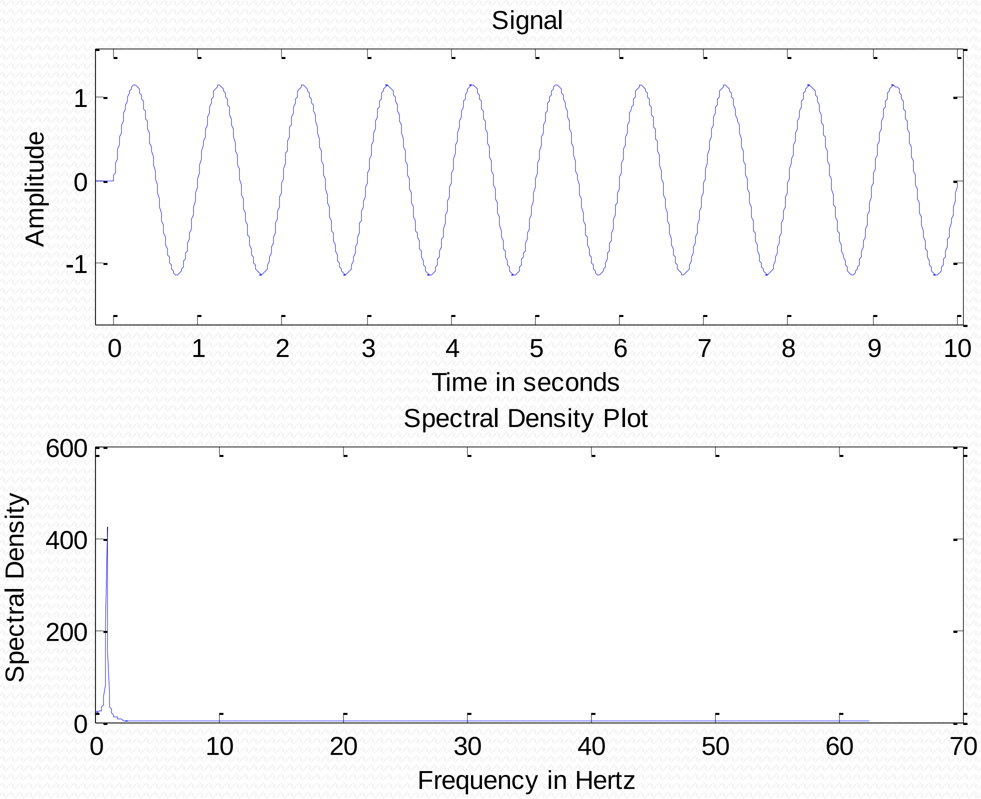
Zero Pole Filter Design

- The simulation shows we did the job
  - Spectrum before the filter



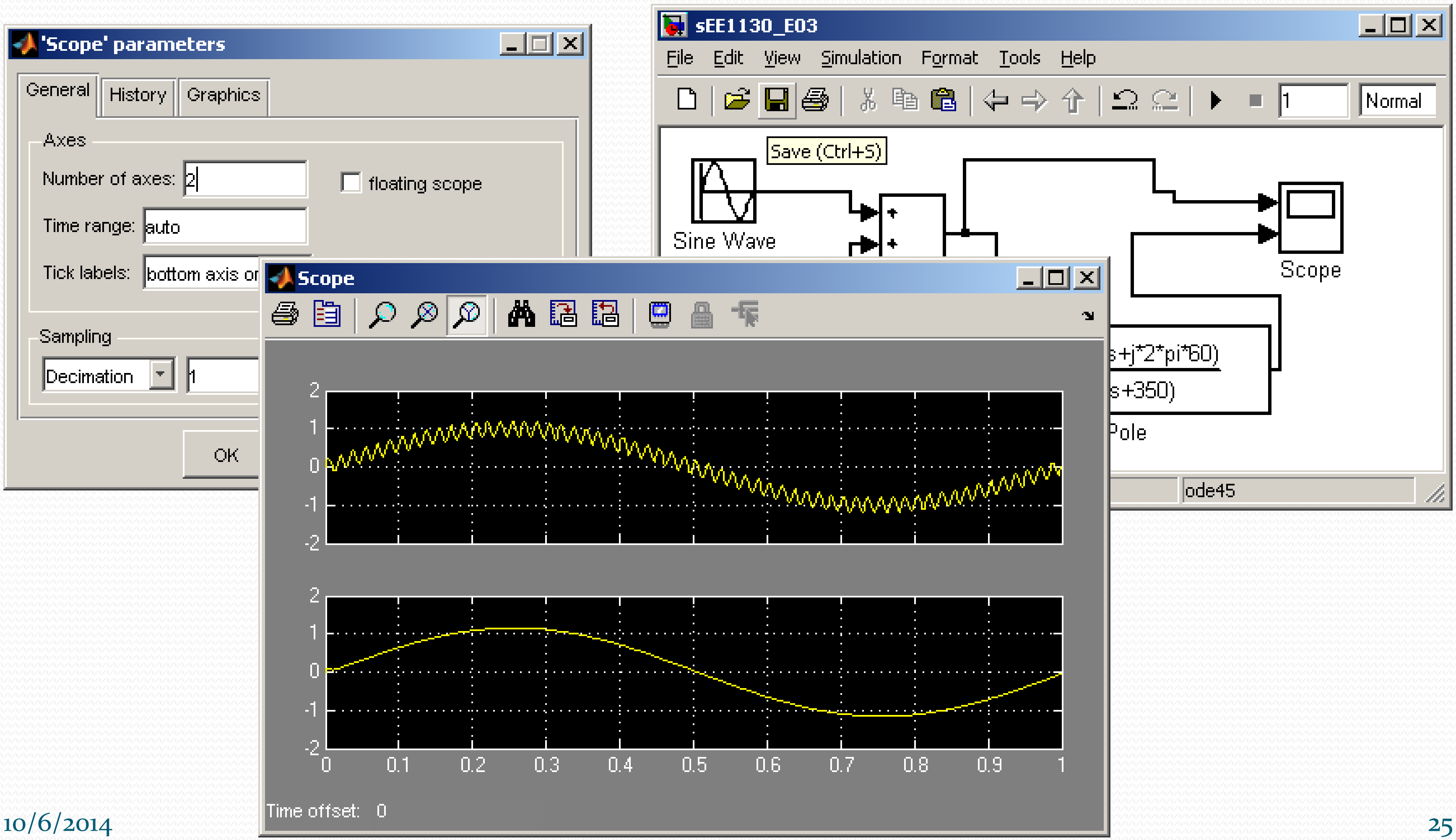
Simulink: Signal Processing.

- The simulation shows we did the job:
  - Spectrum after the filter



Simulink: Signal Processing.

- We notice the dark trace is completely clean of noise. We could add another trace to the scope and see both signals separated:



Implementation

- Once the simulation shows we solved the problem, we need to implement the Electrical Circuit.
- In order to do that, we need to modify the Transfer Function in a sum of simpler Transfer Functions of the type:

$$H_{simple}(s) = \frac{G}{(\tau s + 1)}$$

- This is done with Partial Fraction Expansion:

$$H(s) = \frac{s^2 + 142120}{(s+340)(s+360)} = \frac{R_1}{s+340} + \frac{R_2}{s+360}$$

- Matlab calculate the residues very fast:

Implementation

- Matlab calculate the residues very fast:

$$H(s) = \frac{s^2 + 142120}{(s+340)(s+360)} = \frac{-1.35861*10^4}{s+360} + \frac{1.2886*10^4}{s+340}$$

Implementation

- One more modification yields:

$$H(s) = \frac{-1.35861*10^4}{s+360} + \frac{1.2886*10^4}{s+340}$$

$$H(s) = \frac{-37.7}{\frac{1}{360}s+1} + \frac{37.9}{\frac{1}{340}s+1}$$

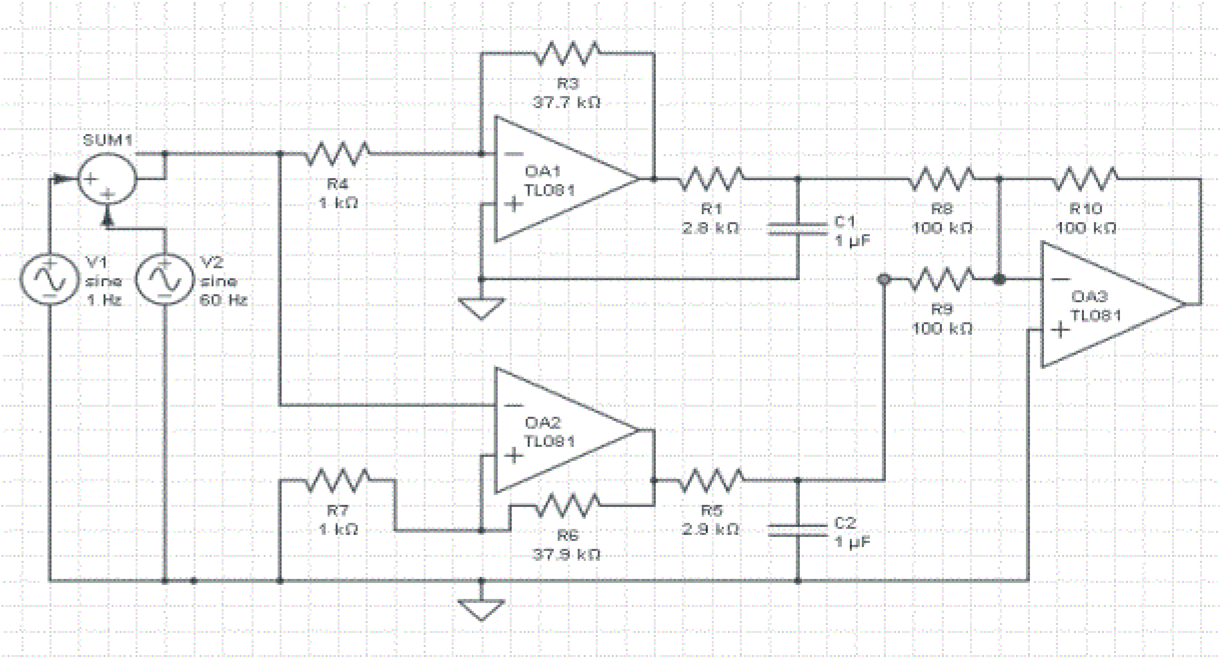
- Each term correspond to a RC circuit:

$$H_{simple}(s) = \frac{G_1}{(R_1C_1s+1)} + \frac{G_2}{(R_2C_2s+1)}$$

Implementation

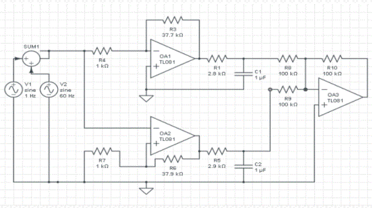
- Implementation:

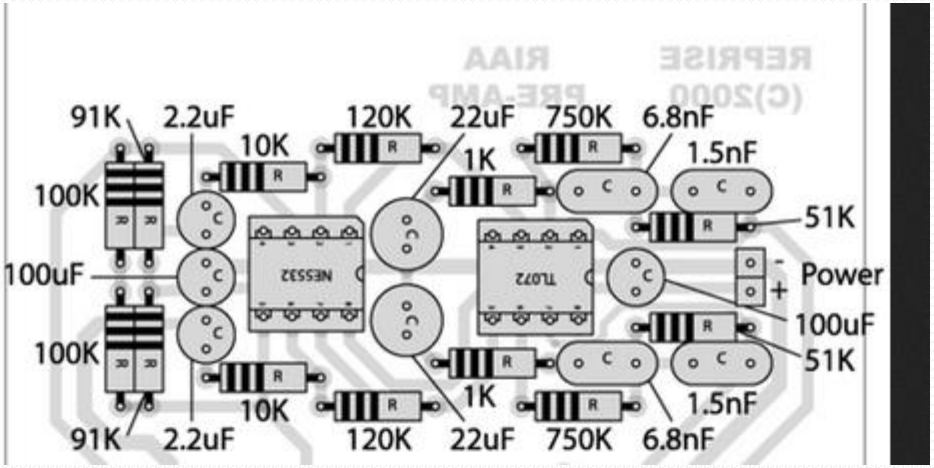
$$H(s)=\frac{-37.7}{\frac{1}{360}s+1}+\frac{37.9}{\frac{1}{340}s+1}$$



Layout

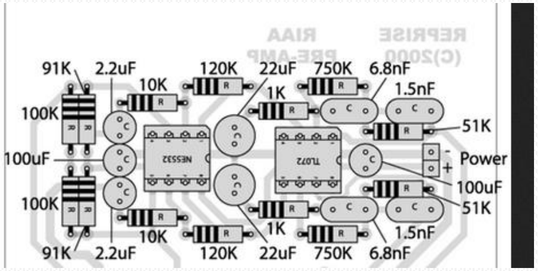
- From the Electrical Schematics we build the physical layout:
- We obtain something like:





Building and Testing

- From the physical layout:
- we build the
  PCB (Printed Circuit Board)



- We solder the components.
- Solder the cables.

- Then we test!!!

Final Report

- We generate the final report with our findings, to validate that the circuit does what we intended it to do.

End of Class