EE 1130

Freshman Eng. Design for Electrical and Computer Eng.

Class 3

Signal Processing Module (DSP).

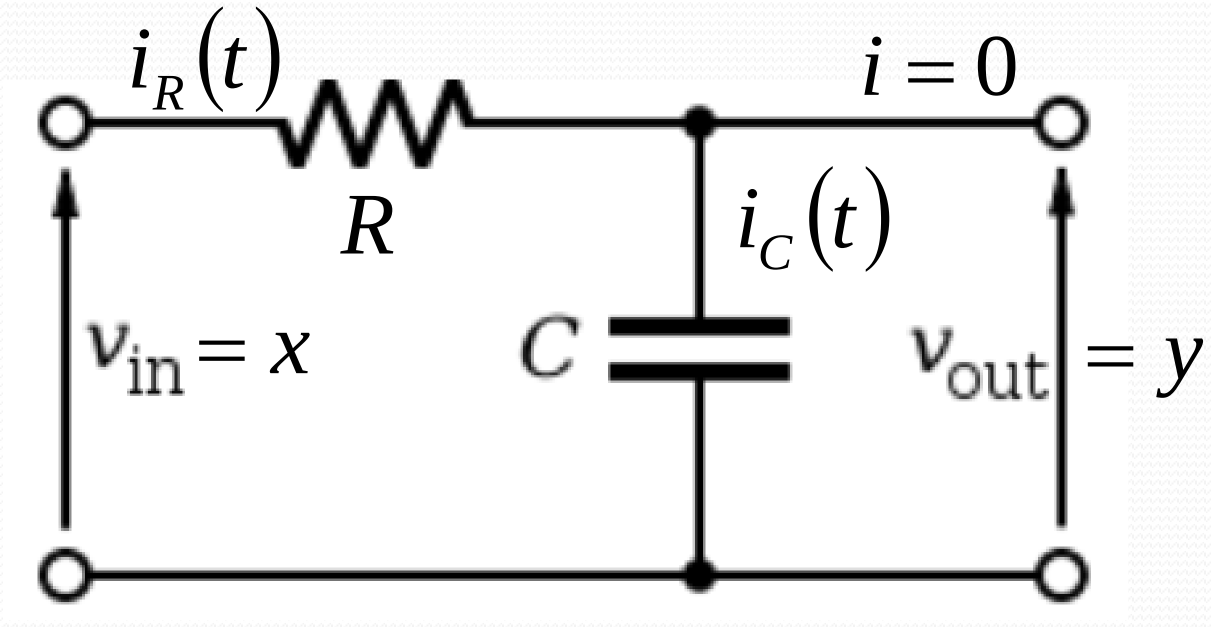
- Differential Equations.
- Simulink: Differential Equations.
- Design of Electrical Circuits with CircuitLab.

Filter Design.

- Last lecture we generated a EKG (Electrocardiogram) signal with noise. We did that to EMULATE OR MODEL something like a real noisy EKG signal.
- Our next objective is to process (Signal Processing) the generated EKG signal to clean it from the noise.
  - We need knowledge in Digital Filters.
  - But before knowledge in Digital Filters, we need to acquire knowledge in Analog Filters.

Differential Equations.

- Any Linear Time Invariant system could be modeled as the solution of a differential equation (DE) .
- In the case of Low Pass RC filter shown in next figure:



- The Differential Equation is:

$$RC\dot{y} + y = x$$

Differential Equations.

- Next is an algebraic equation (instant equation).

$$y(t) = 2x(t)$$

- A differential equation has into account velocities!!!

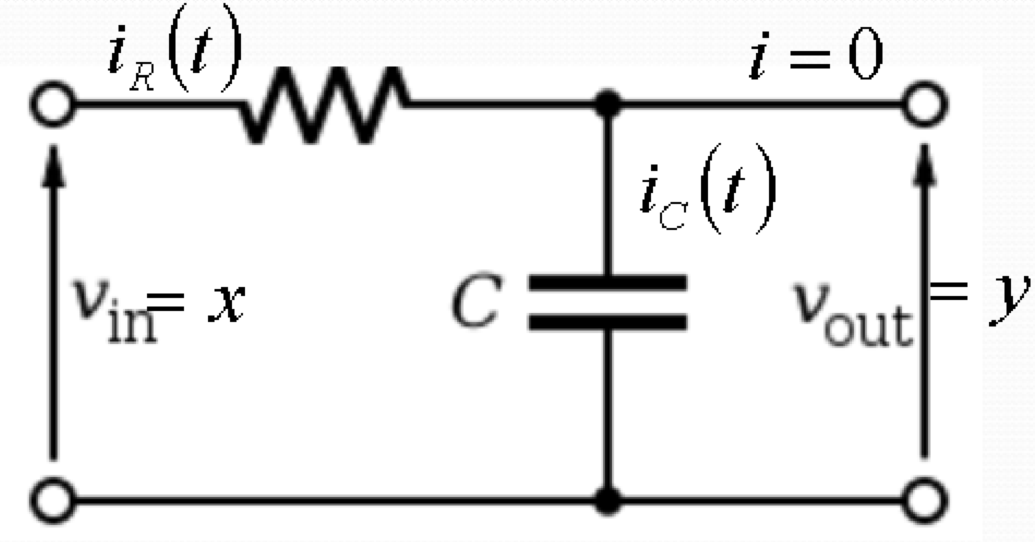
$$K\frac{dy}{dt} + y(t) = 2x(t)$$

- Another way of writing the expression above is:

$$K\dot{y} + y(t) = 2x(t)$$

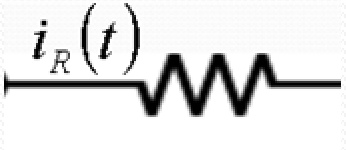
Differential Equations.

- The circuit analysis is shown in next figure:
  - x or $x(t)$ is the input voltage, also called v_{in}.
  - y or y(t) is the output voltage, also called v_{out}.
  - Our objective is to find a mathematical expression that links $y(t)$ with $x(t)$. Also called input-output relation or function.



Differential Equations.

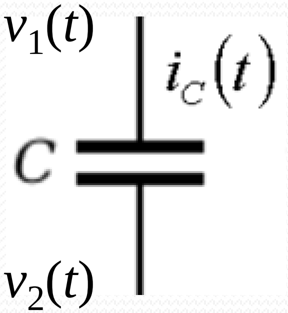
- First of all, we need to know the voltage current relation of each component:
  - Resistor: the relation IV is called Ohm Law!!
  - $v_1(t)$ is the voltage at the left side of R
  - $v_2(t)$ is the voltage at the right side of R
  - $i_R(t)$ is the current flow from left to right side of R. Current is measured in Amperes.
  - The value of the resistor is R (measured in Ohms).
  - Voltage is measured in Volts.
  - Ohms law: $i_R(t) = (v_1(t) - v_2(t))/R = v_R(t)/R$
  - We define $v_R(t) = v_1(t) - v_2(t)$



$v_1(t)$ $v_2(t)$

Differential Equations.

- Secondly, we need to know the voltage current relation the Capacitor:
  - $v_1(t)$ is the voltage at the upper side of C
  - $v_2(t)$ is the voltage at the lower side of C
  - $i_C(t)$ is the current flow from top to bottom of C. Current is measured in Amperes.
  - The value of the capacitor is C (measured in Farads).
  - Voltage is measured in Volts.



- Capacitor stores voltage!!!
  - IV law: $v_1(t) = v_2(t)$ + accumulation of $i_C(t)$ /C
  - $v_1(t)$- $v_2(t)$ = integral from 0 to t of $i_C(t)$ /C
  - We define $v_C(t) = v_1(t)- v_2(t)$

$$v_C(t) = \frac{1}{C}\int_0^t i_C(t)dt$$

Differential Equations.

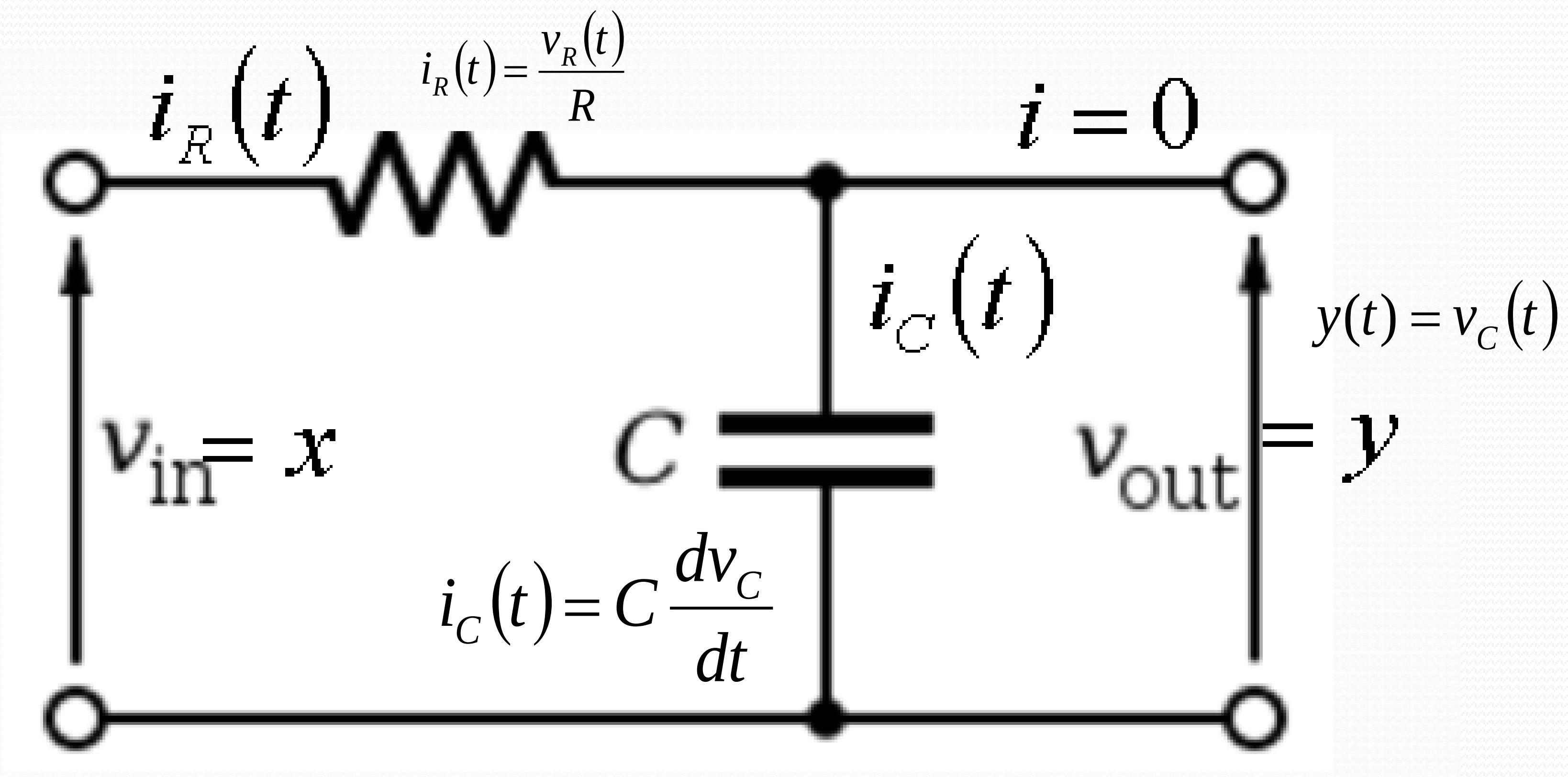
- The following expressions are equivalent:

$$v_C(t) = \frac{1}{C}\int_0^t i_C(t)\,dt$$

$$C\frac{dv_C}{dt} = i_C(t)$$

Differential Equations.

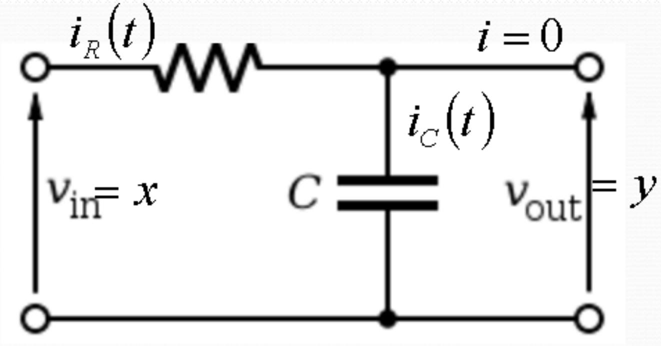
- The circuit analysis is shown in next figure:



Differential Equations.

- To obtain input output relation we will use the Kirchoff laws:
  - KVL: voltage delivered = voltage consumed!!

$$x(t) = v_R(t) + v_C(t)$$

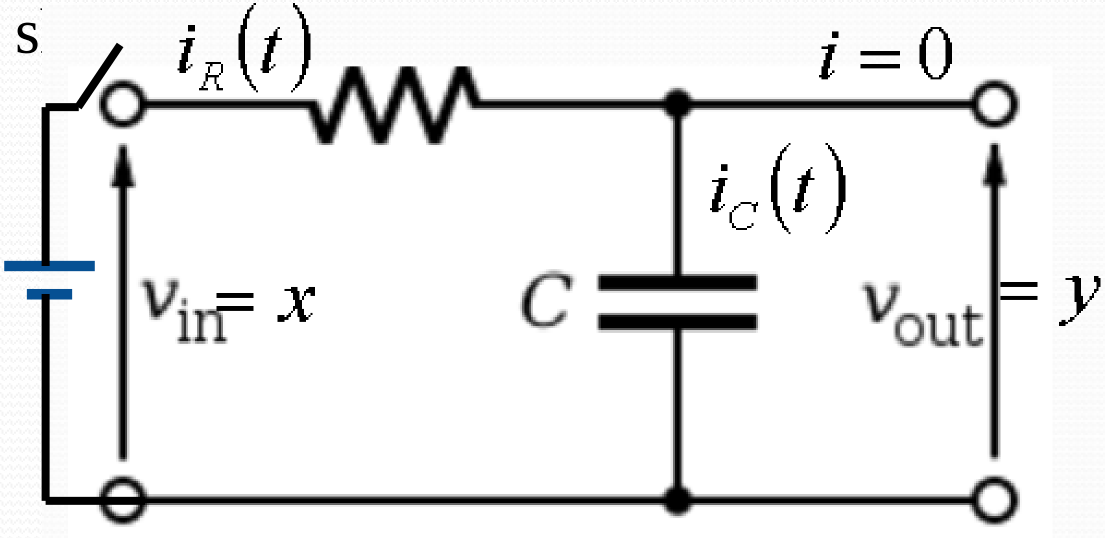


  - KCL: current entering in a node = current exiting a node!!

$$i_R(t) = i_C(t) + 0$$

Differential Equations.

- The circuit analysis is s



$$x(t) = v_R(t) + v_C(t)$$

$$x(t) = i_R(t)R + y(t)$$

$$x(t) = \left(C\frac{dy}{dt}\right)R + y(t)$$

$$RC\frac{dy}{dt} + y(t) = x(t)$$

$$RC\dot{y} + y(t) = x(t)$$

Differential Equations.

$$RC\dot{y} + y = x$$

- Where y with the dot is the first derivative of $y(t)$ and x is $x(t)$. R and C are the values of the Resistor and Capacitor respectively.
- The Differential Equation could be simulated with Simulink.
- However, the Differential Equation must be modified to an Integral Equation, since integrator blocks are more used than derivative blocks.

$$\int \left(RC\dot{y} + y\right)dt = \int x\,dt$$

Differential Equations.

- The integral is linear:

$$RC\int \dot{y}\,dt + \int y\,dt = \int x\,dt$$

$$RCy = \int x\,dt - \int y\,dt$$

$$y = \frac{1}{RC}\int (x - y)dt$$

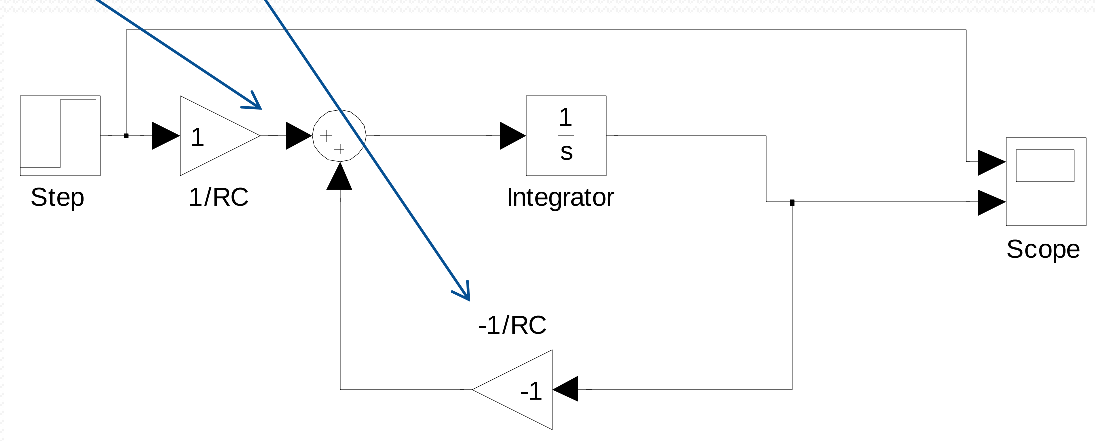
$$y = \int \left(\frac{1}{RC}x - \frac{1}{RC}y\right)dt$$

Simulink: Differential Equations.

- The block diagram could be implemented from this equation:

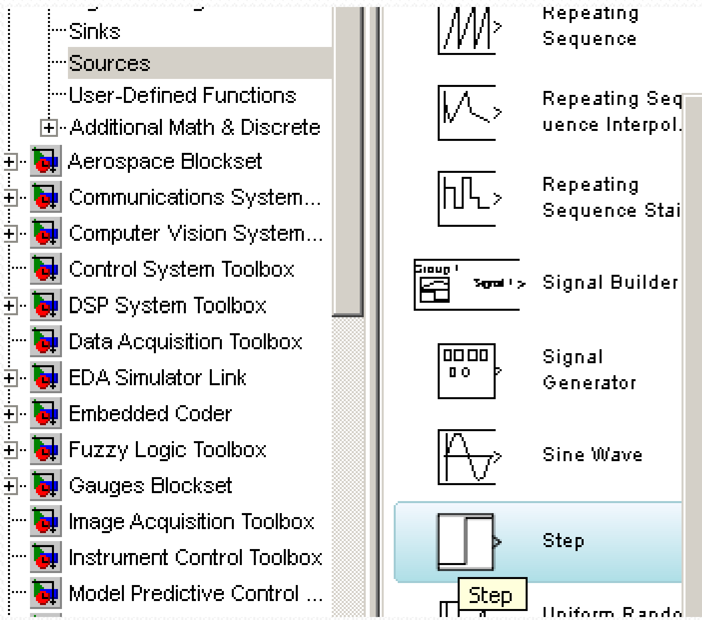
$$y = \int \left(\frac{1}{RC} x - \frac{1}{RC} y \right) dt$$

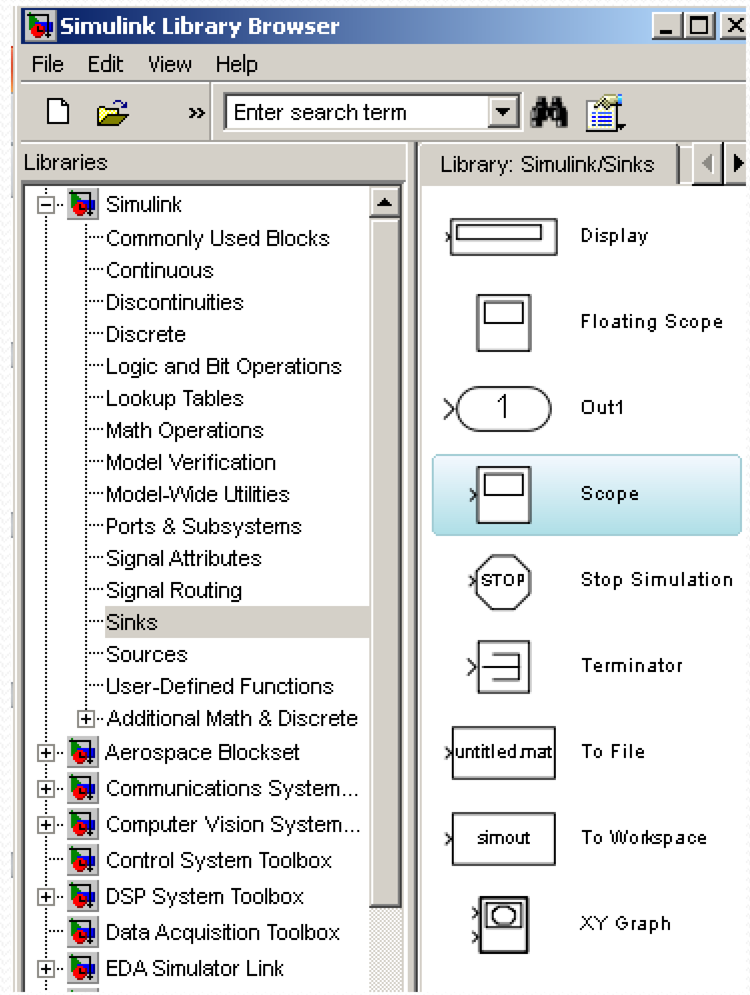
$$\frac{1}{RC} x - \frac{1}{RC} y$$



Simulink: Differential Equations.

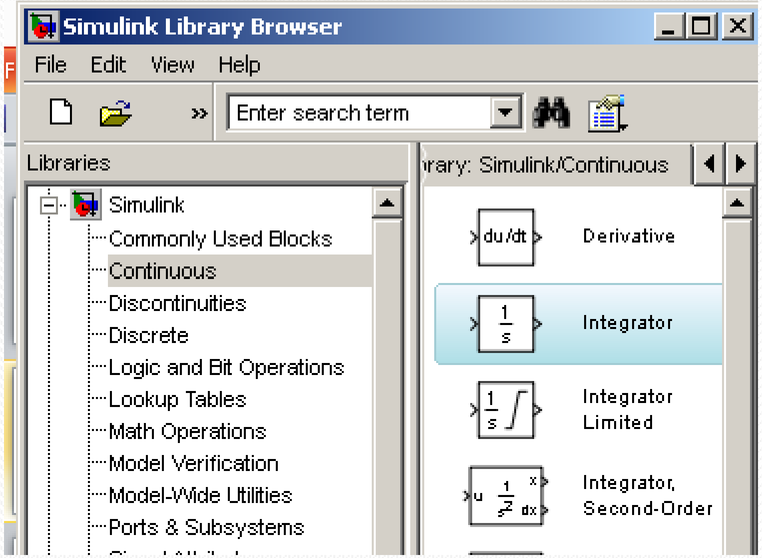
- To insert the Step and Scope blocks we do:





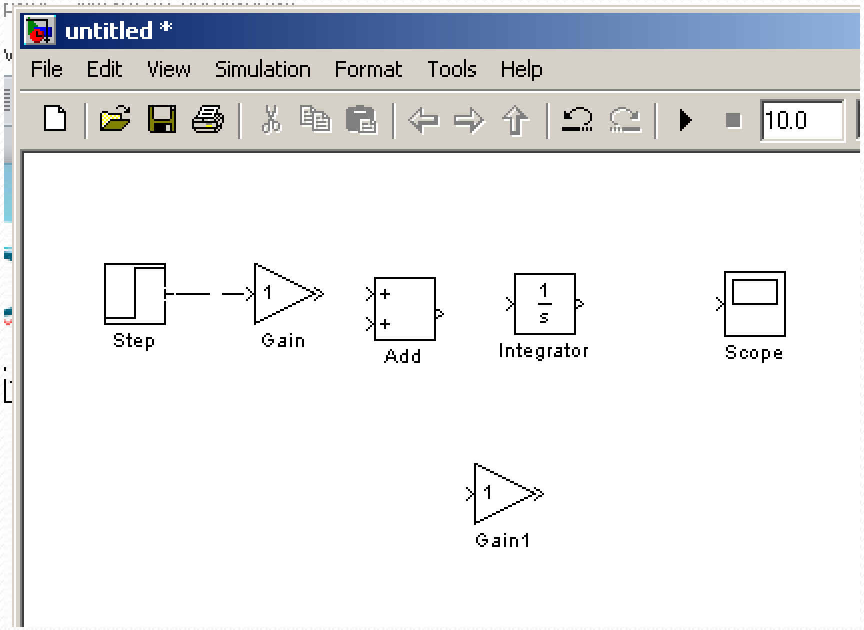
Simulink: Differential Equations.

- To insert the Integrator block we do:

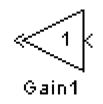


Simulink: Differential Equations.

- Once all elements in the Model, we make the connections:



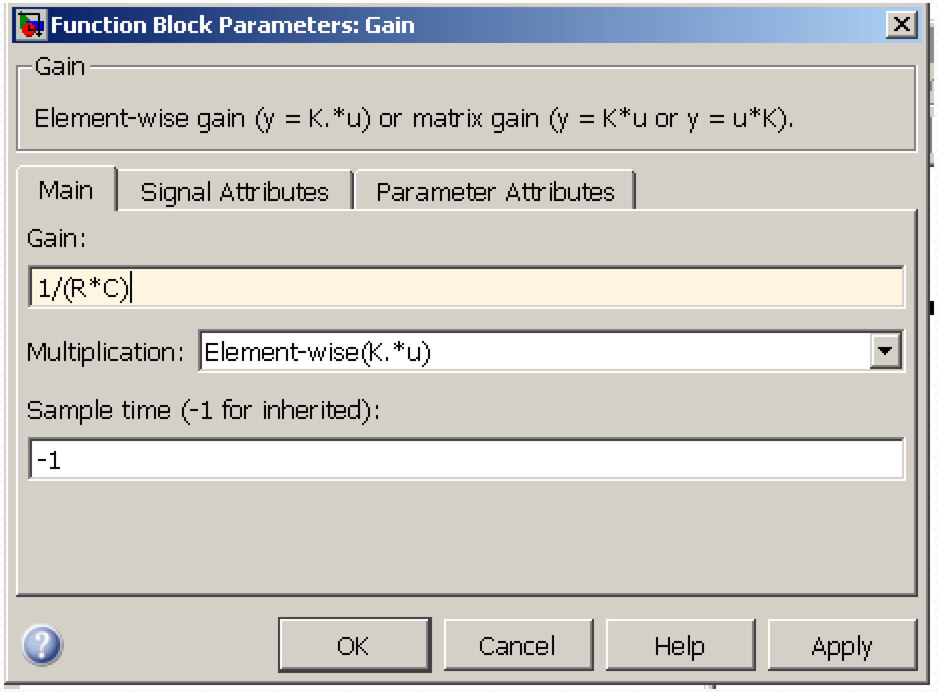
- To flip the Gain1 block we type control+I

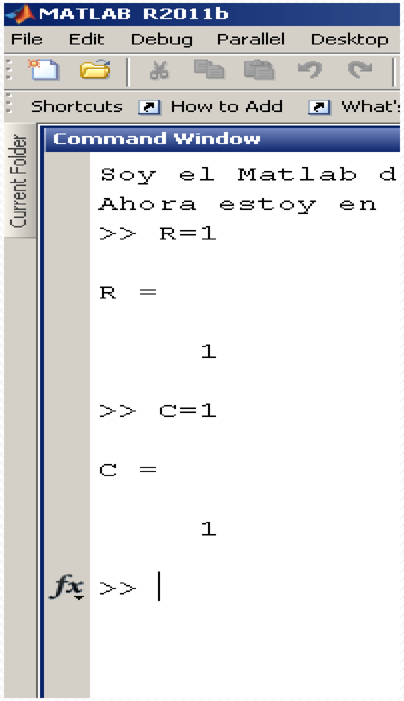


Simulink: Differential Equations.

- Double click on each gain block and change the 1 to 1/(R*C) at the first block and -1/(R*C) at the second.

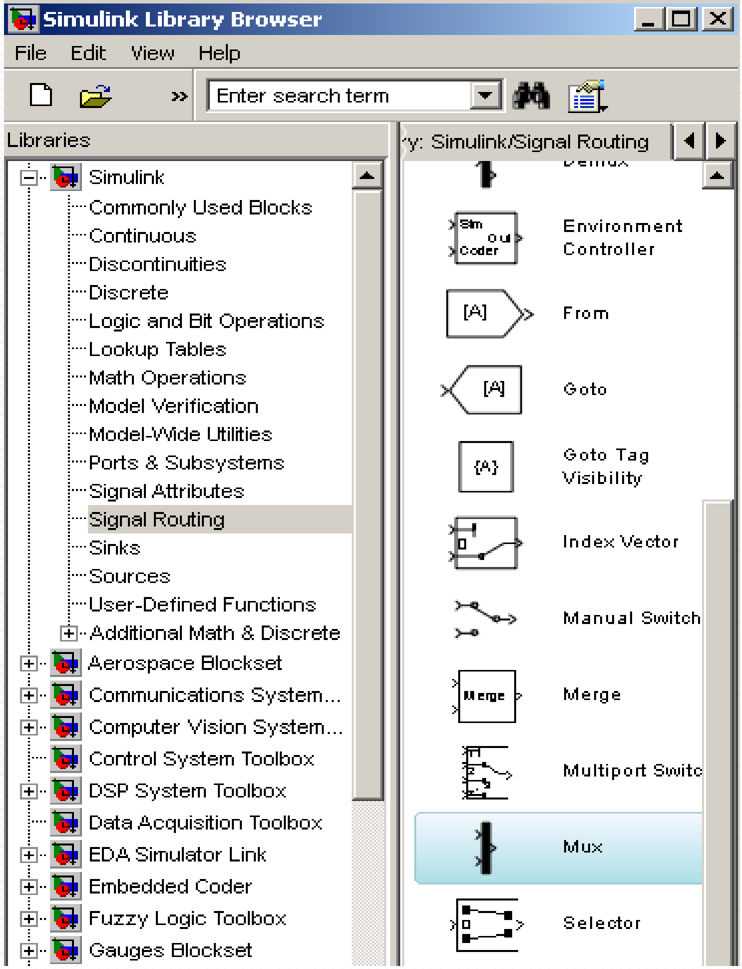
- Type C=1 and R=1 at the command window to define the variables R and C.





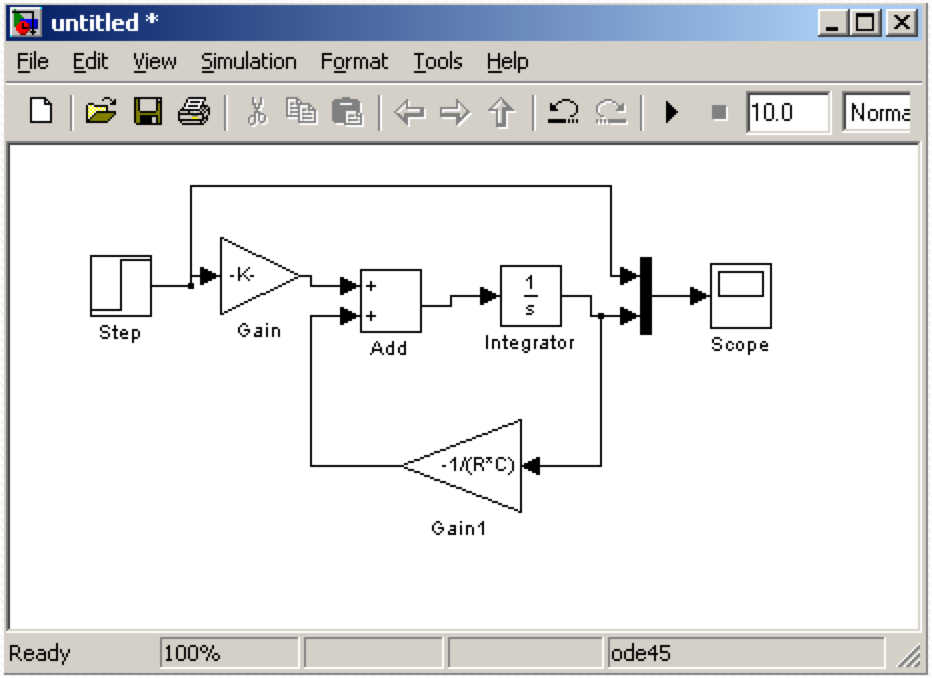
Simulink: Differential Equations.

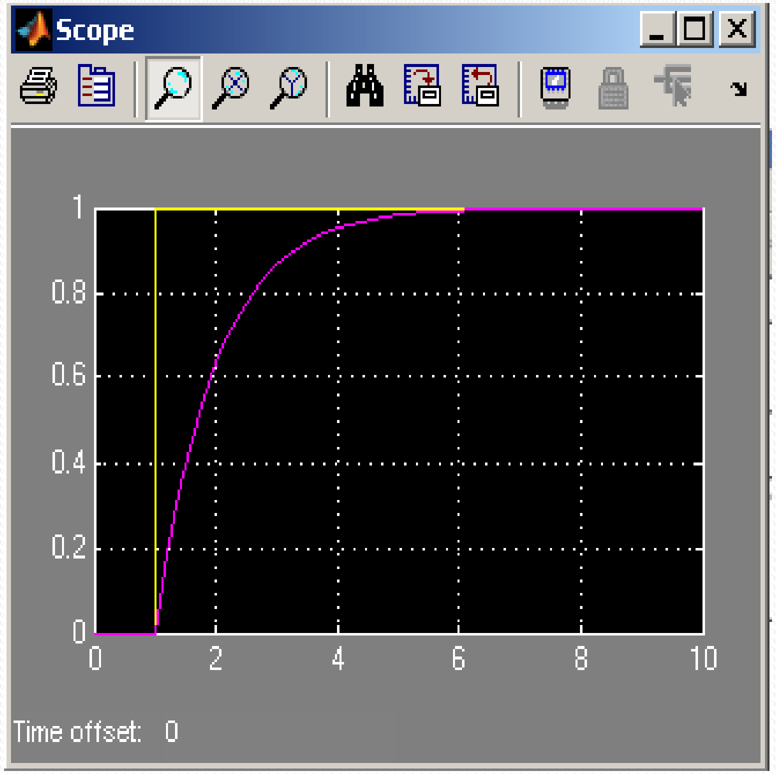
- To see both traces in one scope we add the MUX.



Simulink: Differential Equations.

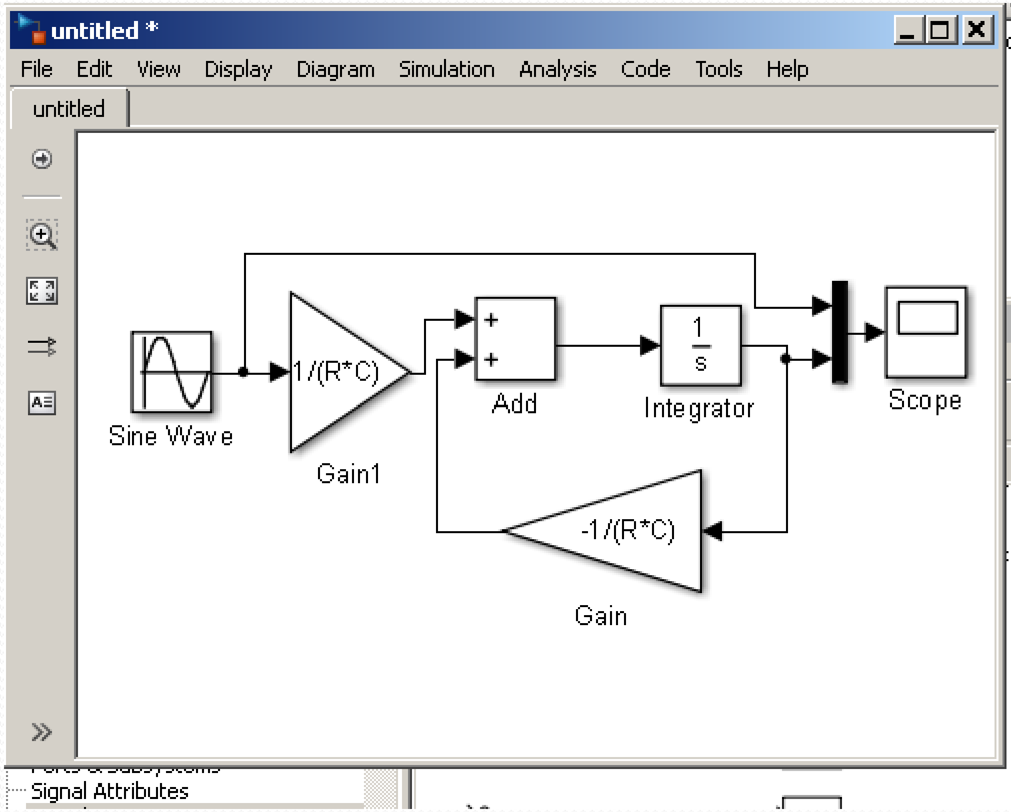
- Once all connected, variables defined in command window, we hit play and double click on the scope block to open the scope screen.

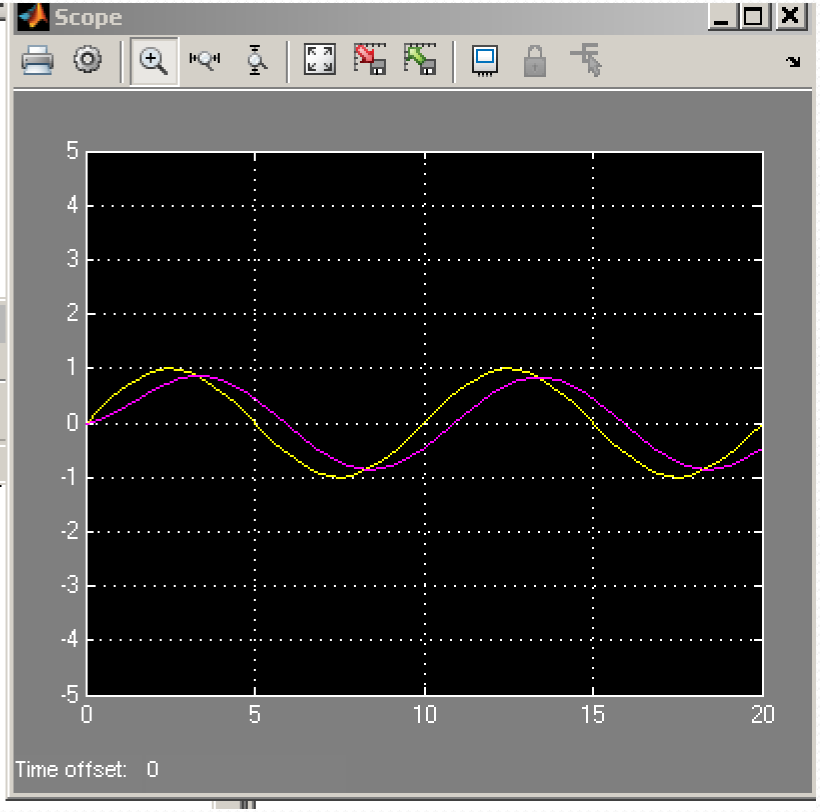




Simulink: Differential Equations.

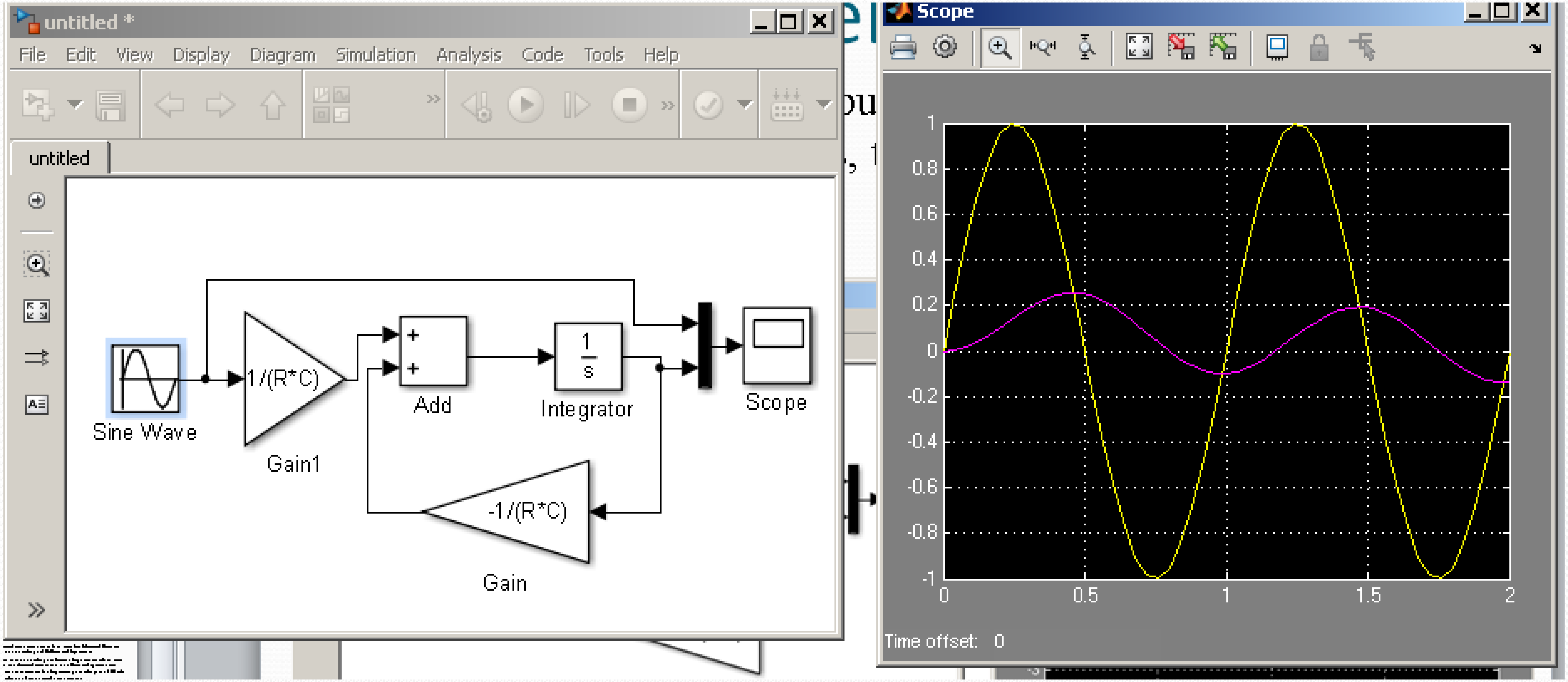
- When using sinusoidal inputs the circuit attenuates larger frequencies. In the figures, the frequency is 1/10Hz.





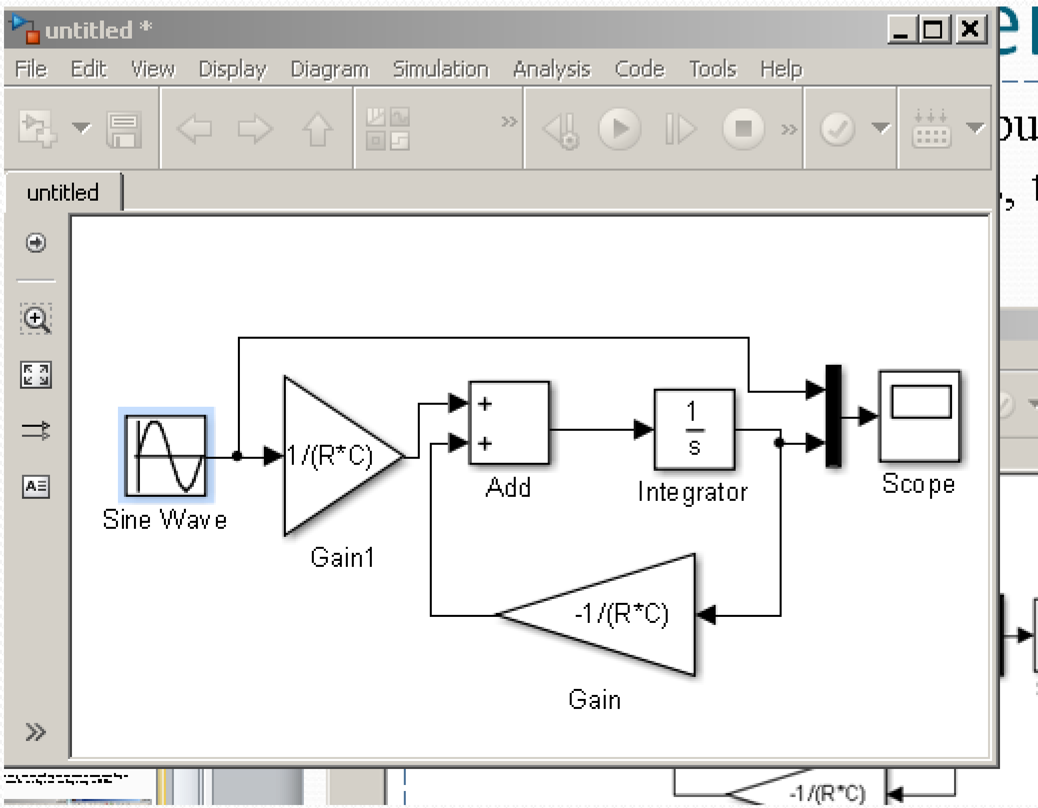
Simulink: Differential Equations.

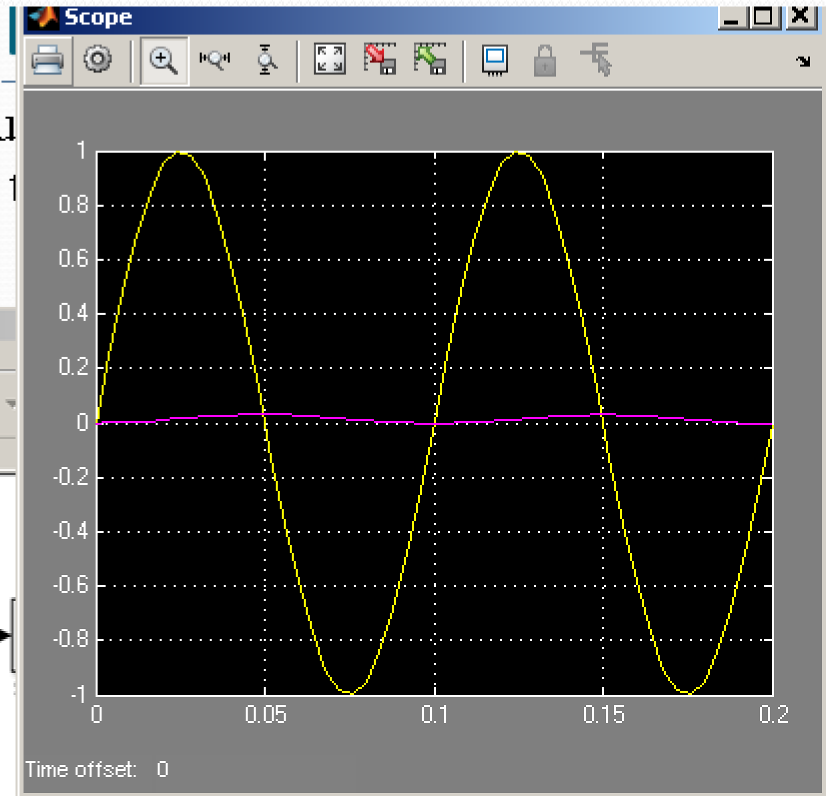
- When using sinusoidal inputs the circuit attenuates larger frequencies. In the figures, the frequency is 1Hz.



Simulink: Differential Equations.

- When using sinusoidal inputs the circuit attenuates larger frequencies. In the figures, the frequency is 10Hz.



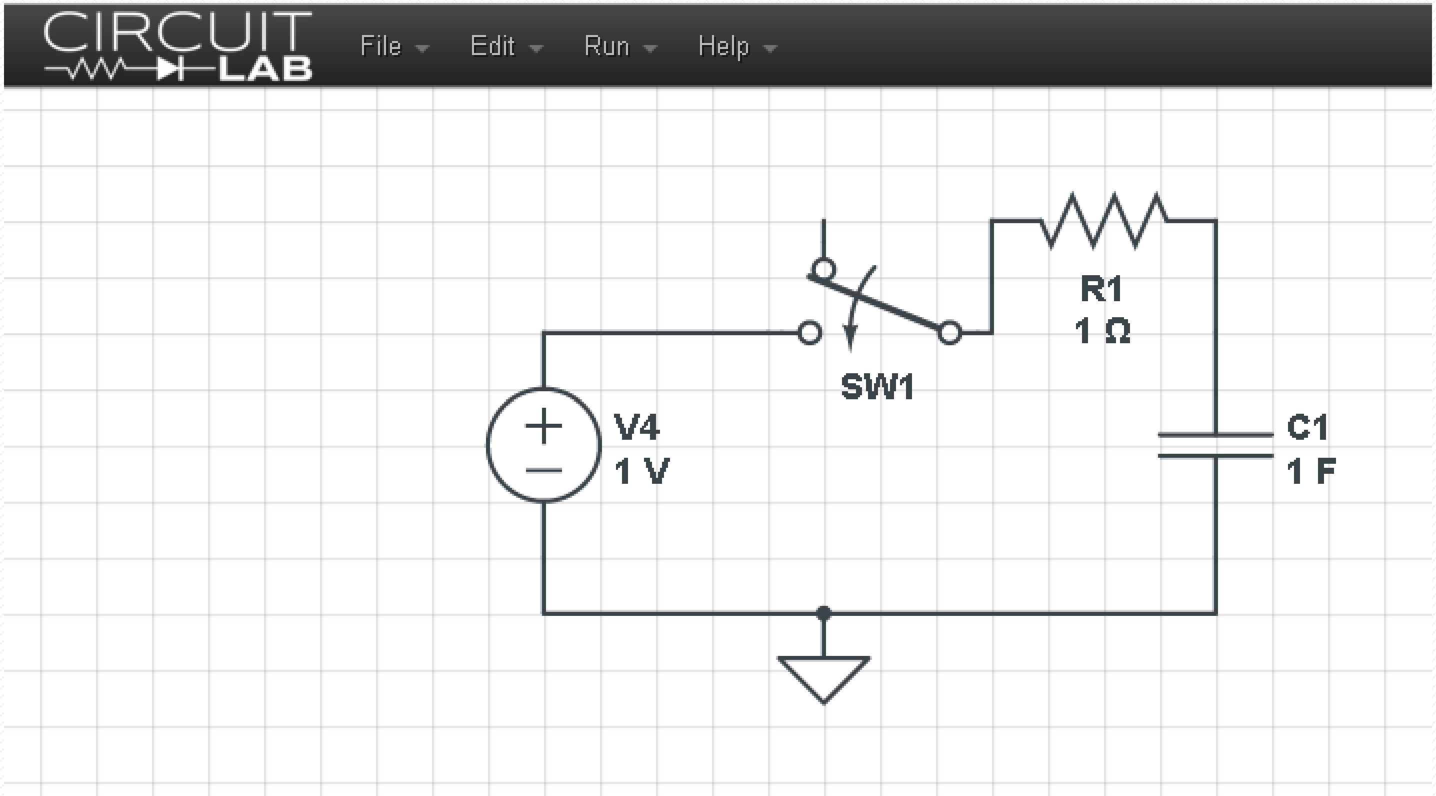


Simulation of Electrical Circs.

- Electrical technicians are able to simulate Electrical Circuits without knowledge of Differential Equations.
- However the knowledge is limited. Engineers are able to know both the electrical circuit and the differential equation that governs that circuit. The Engineering knowledge is a TOTAL knowledge of the physical phenomenon.

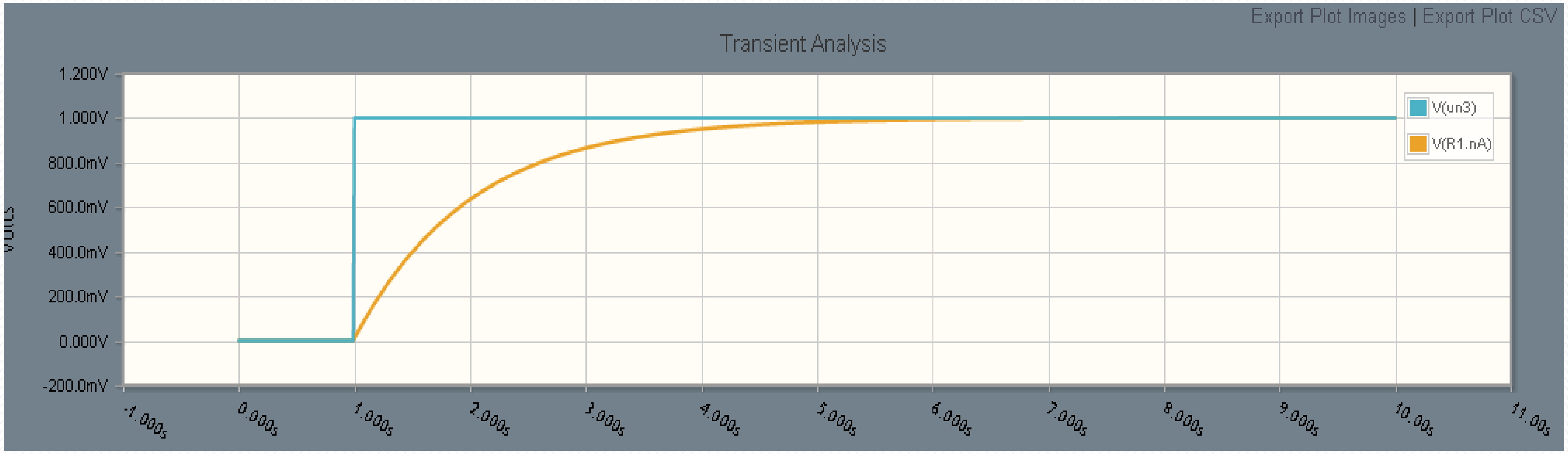
Simulation of Electrical Circs.

- To simulate Electrical circuits we use a online simulator:
  - https://www.circuitlab.com/
- The simulation circuit is shown in next figure!!

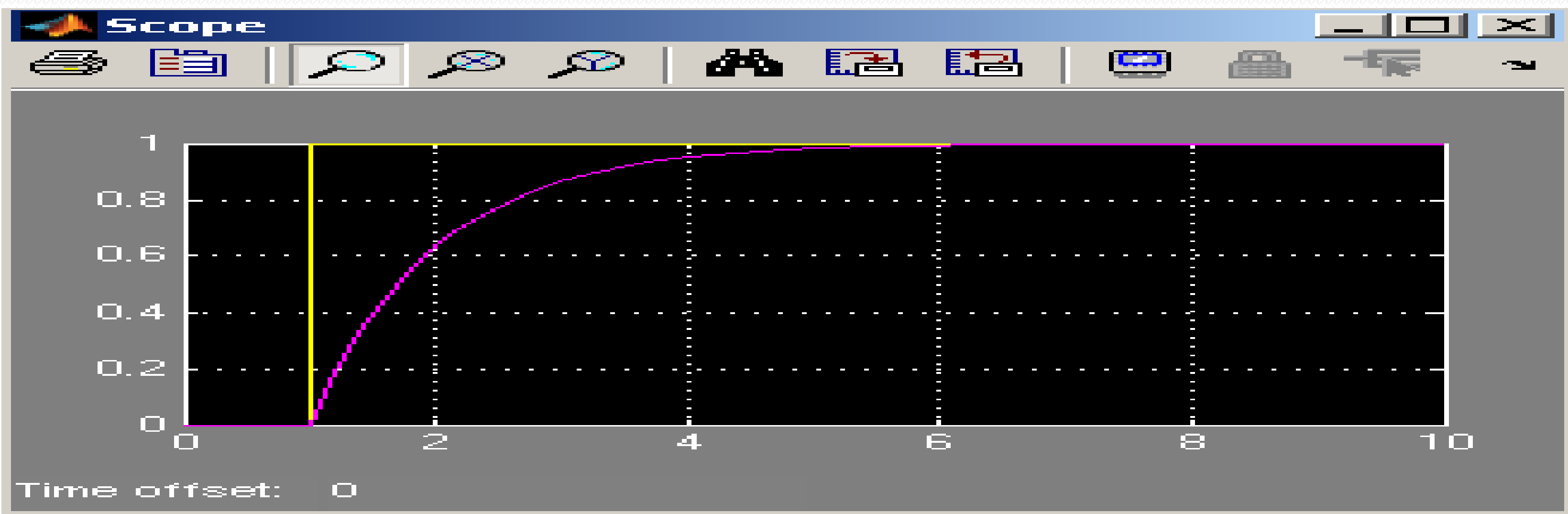


Simulation of Electrical Circs.

- Running simulation we have:



- Same as the result obtained from the Differential Eq. in Matlab



End of Class