

EE 1130

Freshman Eng. Design for Electrical and Computer Eng.

Class 3

Signal Processing Module (DSP).

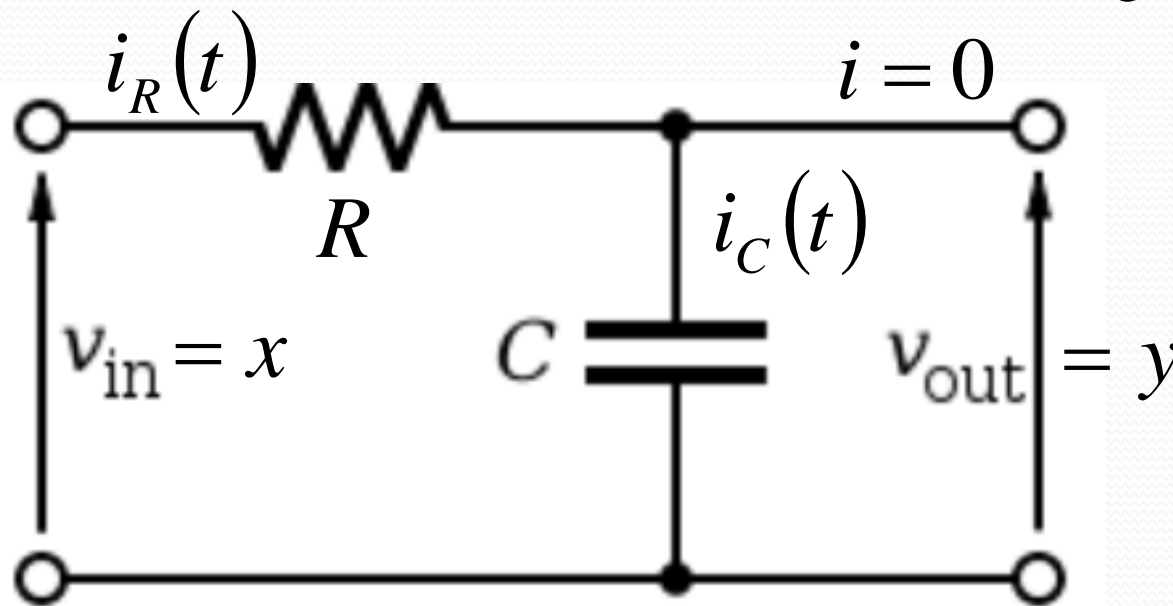
- Differential Equations.
- Laplace Transform. Transfer Function.
- Simulink with Transfer Functions. Zeros, Poles.

Filter Design.

- Last lecture we generated a EKG signal with noise. We did that to EMULATE OR MODEL a real noisy EKG signal.
- Our objective is to process (Signal Processing) the generated EKG signal to clean it from the noise.
- We need knowledge in Digital Filters.
- But before knowledge in Digital Filters, we need to acquire knowledge in Analog Filters.

Simulink: Differential Equations.

- Any Linear Time Invariant system could be modeled as the solution of a differential equation (DE) .
- In the case of Low Pass RC filter shown in next figure:



- The Differential Equation is:

$$RC\dot{y} + y = x$$

Simulink: Differential Equations.

- Next is an algebraic equation (instant equation).

$$y(t) = 2x(t)$$

- A differential equation has into account velocities!!!

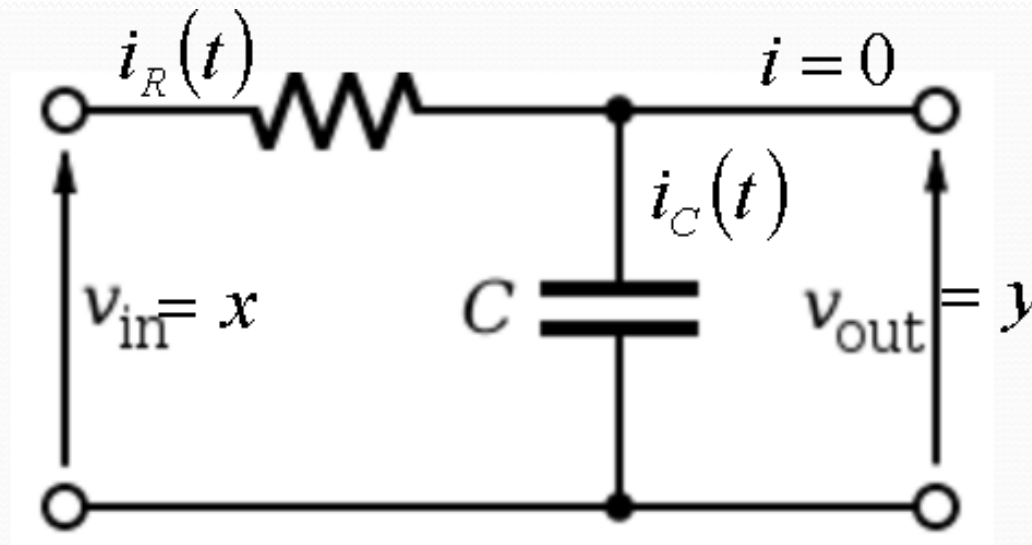
$$K \frac{dy}{dt} + y(t) = 2x(t)$$

- Another way of writing the expression above is:

$$K\dot{y} + y(t) = 2x(t)$$

Simulink: Differential Equations.

- The circuit analysis is shown in next figure:
 - x or $x(t)$ is the input voltage, also called v_{in} .
 - y or $y(t)$ is the output voltage, also called v_{out} .
 - Our objective is to find a mathematical expression that links $y(t)$ with $x(t)$. Also called input-output relation or function.



Simulink: Differential Equations.

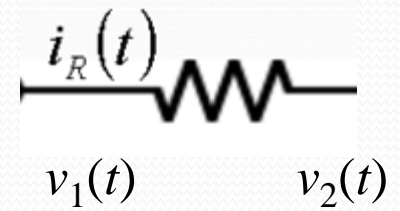
- First of all, we need to know the voltage current relation of each component:

- Resistor: the relation IV is called Ohm Law!!

- $v_1(t)$ is the voltage at the left side of R

- $v_2(t)$ is the voltage at the right side of R

- $i_R(t)$ is the current flow from left to right side of R. Current is measured in Amperes.



- The value of the resistor is R (measured in Ohms).

- Voltage is measured in Volts.

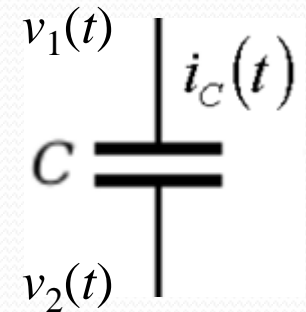
- Ohms law: $i_R(t) = (v_1(t) - v_2(t))/R = v_R(t)/R$

- We define $v_R(t) = v_1(t) - v_2(t)$

Simulink: Differential Equations.

- Secondly, we need to know the voltage current relation the Capacitor:

- $v_1(t)$ is the voltage at the upper side of C
- $v_2(t)$ is the voltage at the lower side of C
- $i_C(t)$ is the current flow from top to bottom of C. Current is measured in Amperes.



- The value of the capacitor is C (measured in Farads).
- Voltage is measured in Volts.
- Capacitor stores voltage!!!
 - IV law: $v_1(t) = v_2(t) + \text{accumulation of } i_C(t) / C$
 - $v_1(t) - v_2(t) = \text{integral from 0 to } t \text{ of } i_C(t) / C$ $v_C(t) = \frac{1}{C} \int_0^t i_C(t) dt$
 - We define $v_C(t) = v_1(t) - v_2(t)$

Simulink: Differential Equations.

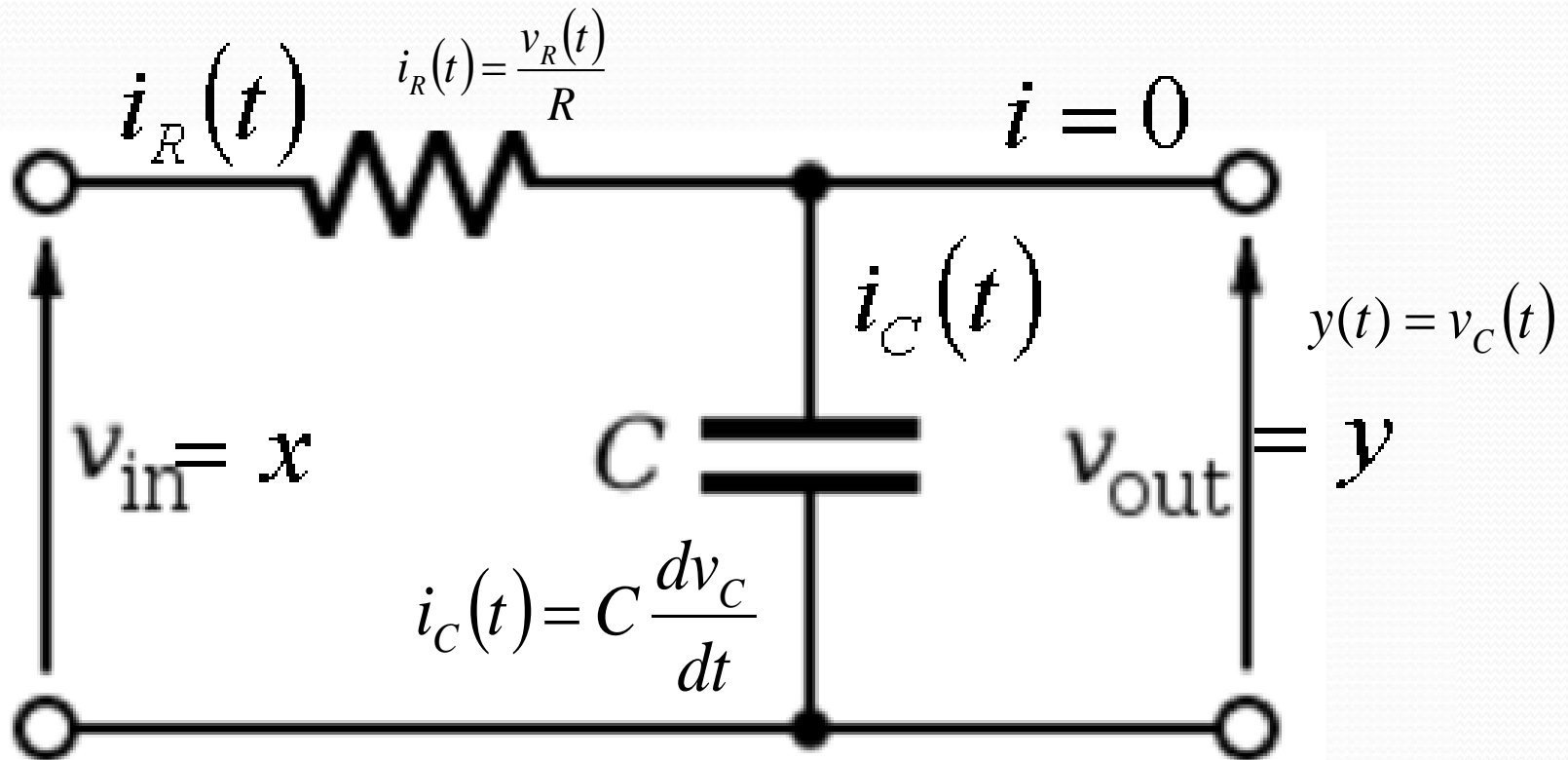
- The following expressions are equivalent:

$$v_C(t) = \frac{1}{C} \int_0^t i_C(t) dt$$

$$C \frac{dv_C}{dt} = i_C(t)$$

Simulink: Differential Equations.

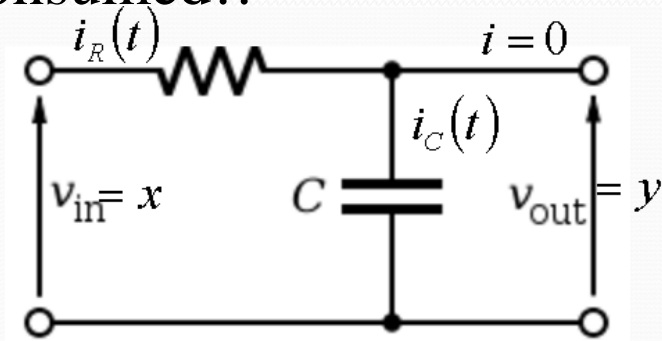
- The circuit analysis is shown in next figure:



Simulink: Differential Equations.

- To obtain input output relation we will use the Kirchoff laws:
 - KVL: voltage delivered = voltage consumed!!

$$x(t) = v_R(t) + v_C(t)$$



- KCL: current entering in a node = current exiting a node!!

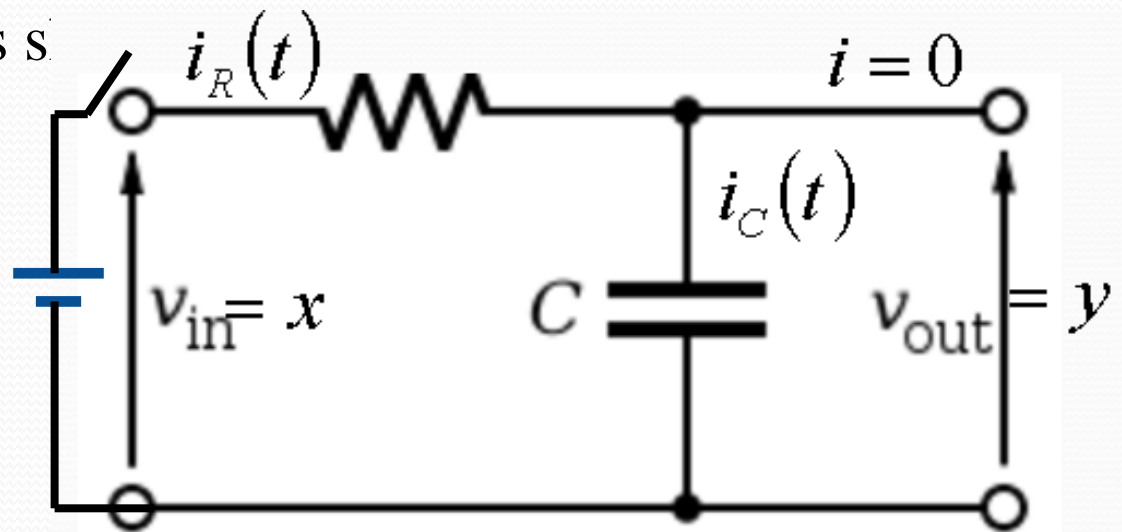
$$i_R(t) = i_C(t) + 0$$

Simulink: Differential Equations.

- The circuit analysis is s

$$x(t) = v_R(t) + v_C(t)$$

$$x(t) = i_R(t)R + y(t)$$



$$x(t) = \left(C \frac{dy}{dt} \right) R + y(t)$$

$$RC \frac{dy}{dt} + y(t) = x(t)$$

$$RC\dot{y} + y(t) = x(t)$$

Simulink: Differential Equations.

$$RC\dot{y} + y = x$$

- Where y with the dot is the first derivative of $y(t)$ and x is $x(t)$. R and C are the values of the Resistor and Capacitor respectively.
- The Differential Equation could be simulated with Simulink.
- However, the Differential Equation must be modified to an Integral Equation, since integrator blocks are more used than derivative blocks.

$$\int (RC\dot{y} + y) dt = \int x dt$$

Simulink: Differential Equations.

- The integral is linear:

$$RC \int \dot{y} dt + \int y dt = \int x dt$$

$$RCy = \int x dt - \int y dt$$

$$y = \frac{1}{RC} \int (x - y) dt$$

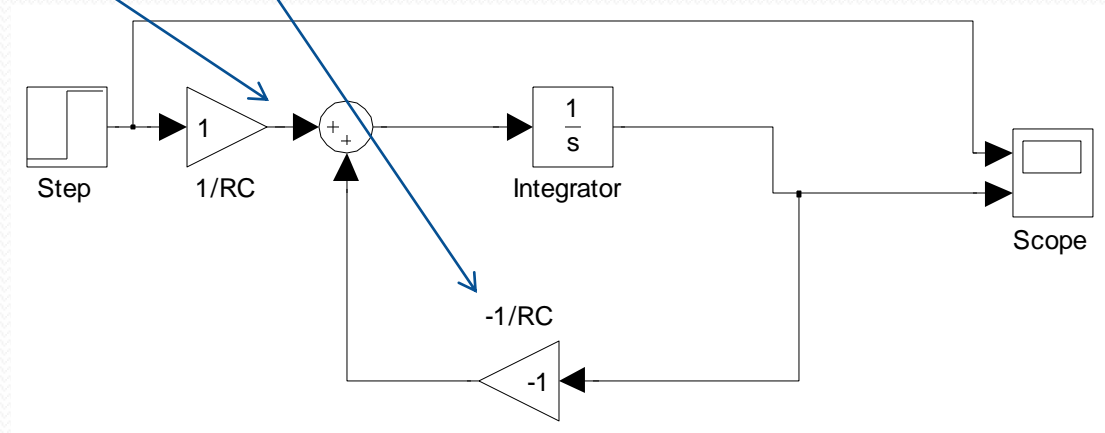
$$y = \int \left(\frac{1}{RC} x - \frac{1}{RC} y \right) dt$$

Simulink: Differential Equations.

- The block diagram could be implemented from this equation:

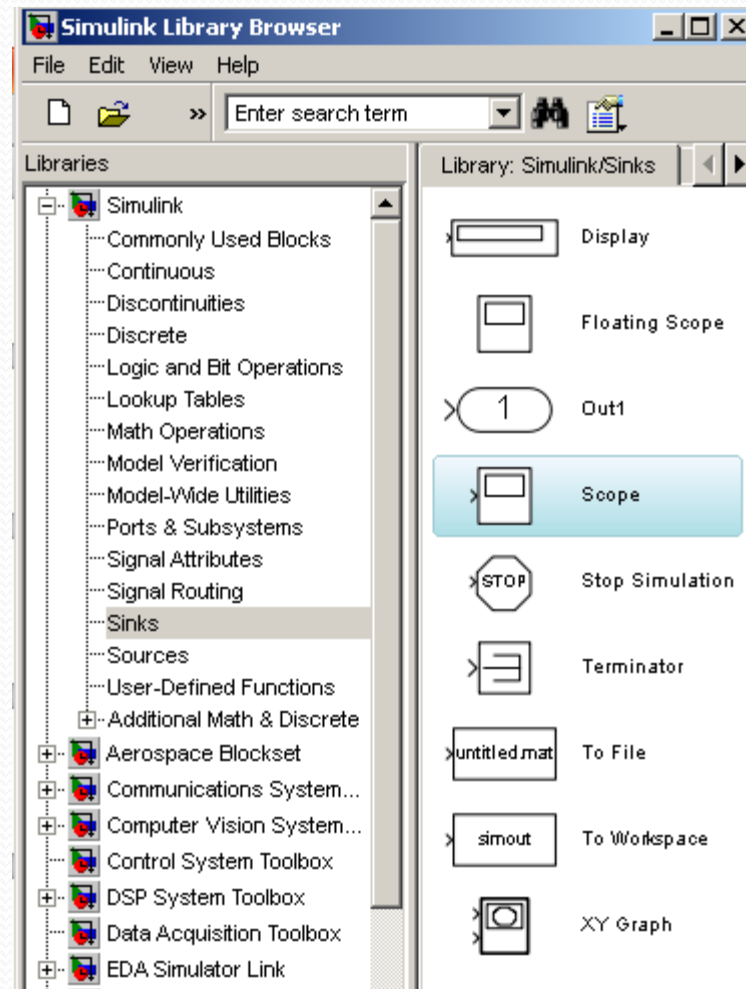
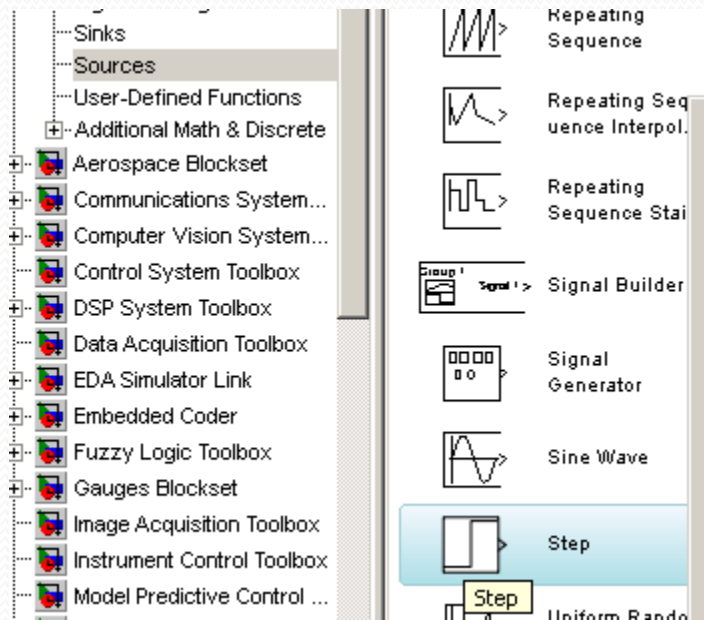
$$y = \int \left(\frac{1}{RC} x - \frac{1}{RC} y \right) dt$$

$$\frac{1}{RC} x - \frac{1}{RC} y$$



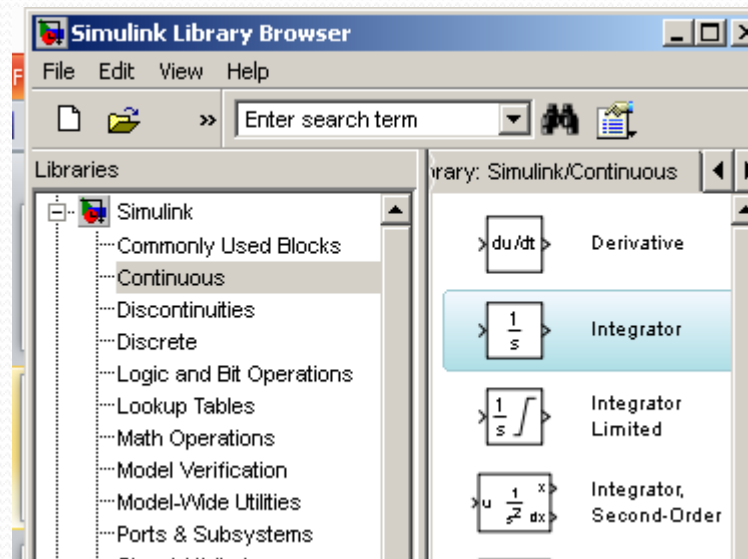
Simulink: Differential Equations.

- To insert the Step and Scope blocks we do:



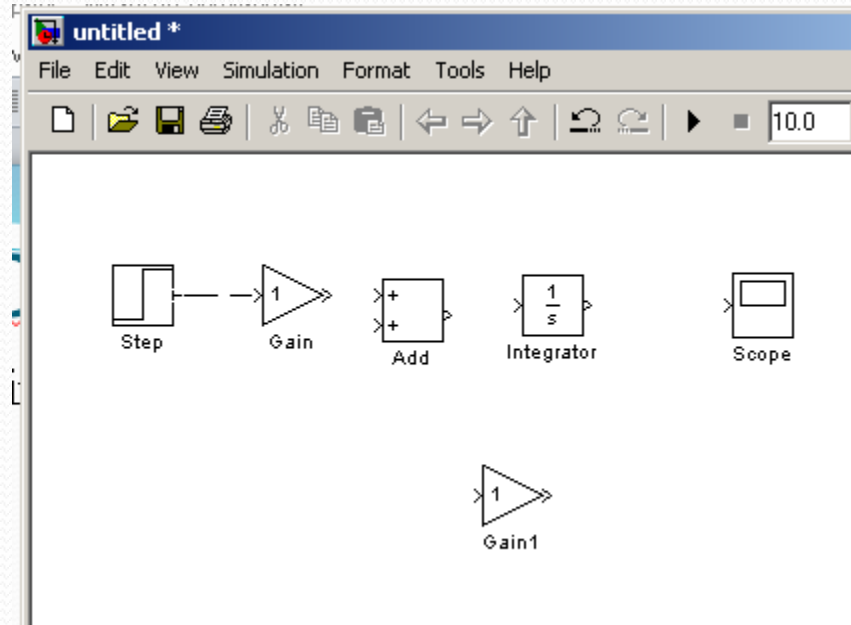
Simulink: Differential Equations.

- To insert the Integrator block we do:



Simulink: Differential Equations.

- Once all elements in the Model, we make the connections:

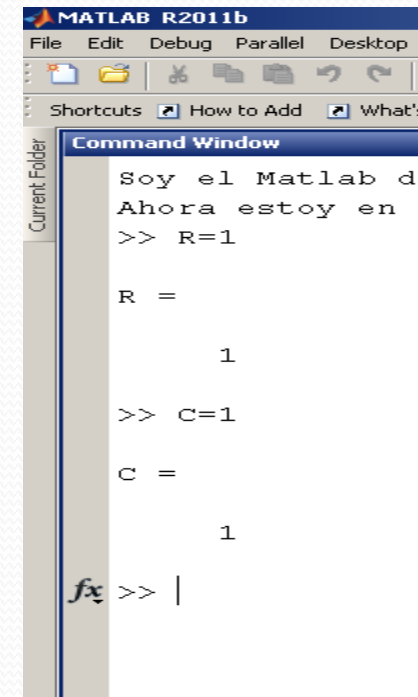
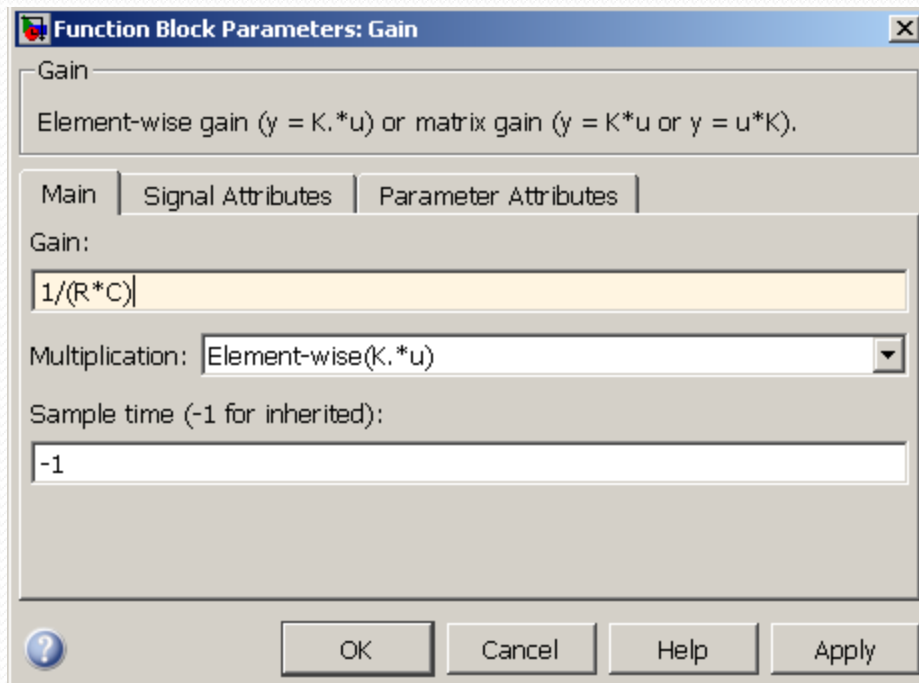


- To flip the Gain1 block we type control+I



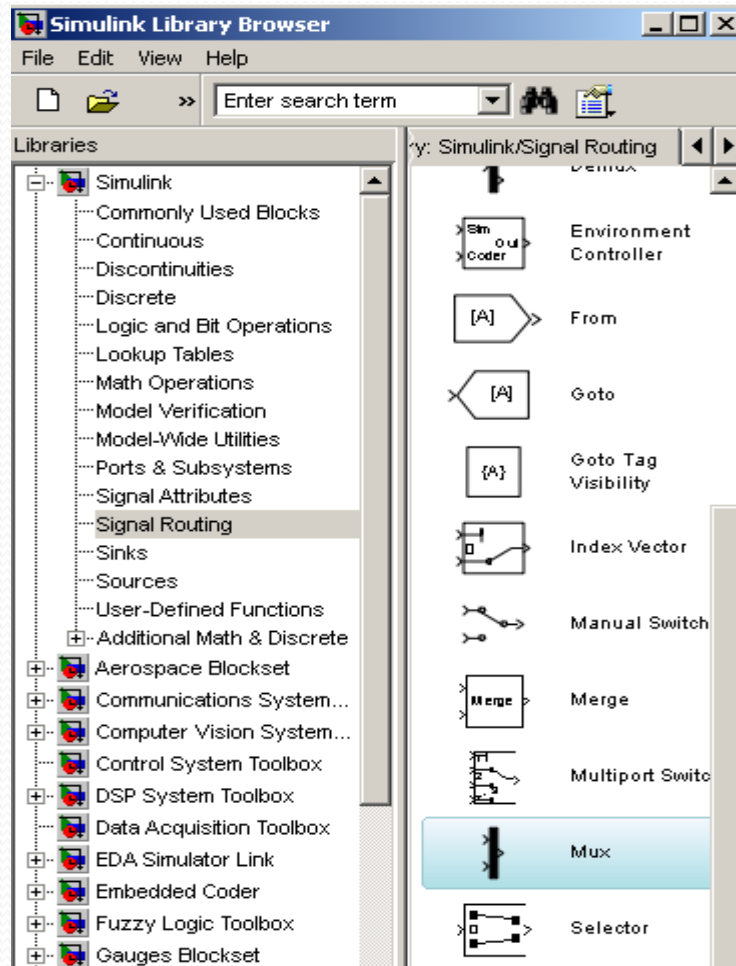
Simulink: Differential Equations.

- Double click on each gain block and change the 1 to $1/(R*C)$ at the first block and $-1/(R*C)$ at the second.
- Type $C=1$ and $R=1$ at the command window to define the variables R and C .



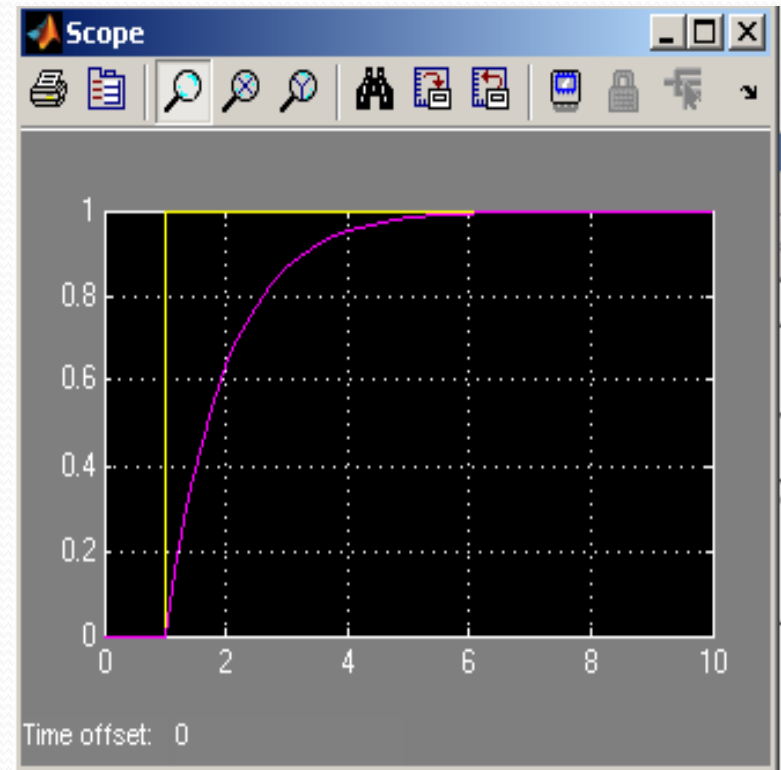
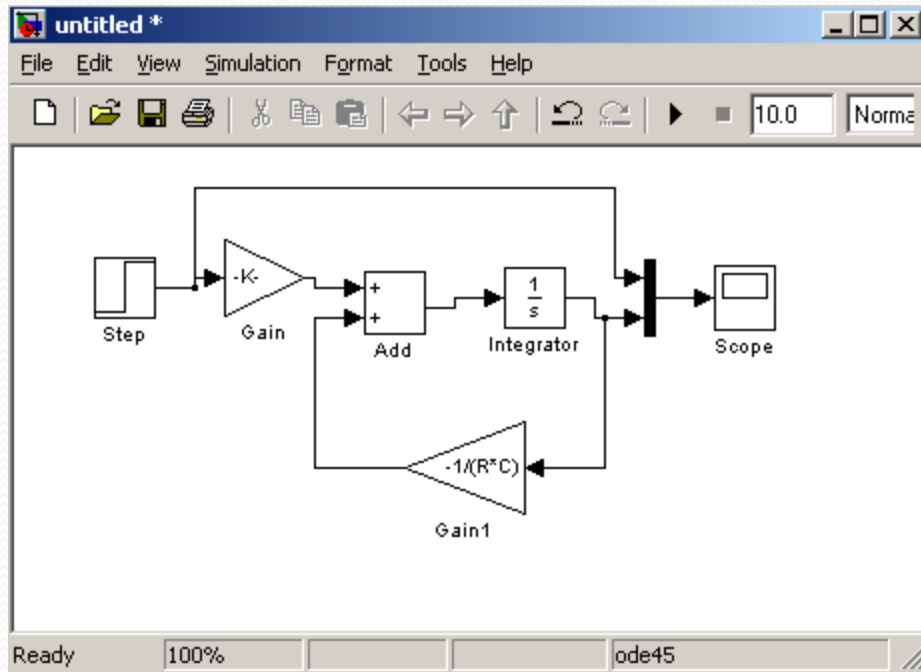
Simulink: Differential Equations.

- To see both traces in one scope we add the MUX.



Simulink: Differential Equations.

- Once all connected, variables defined in command window, we hit play and double click on the scope block to open the scope screen.

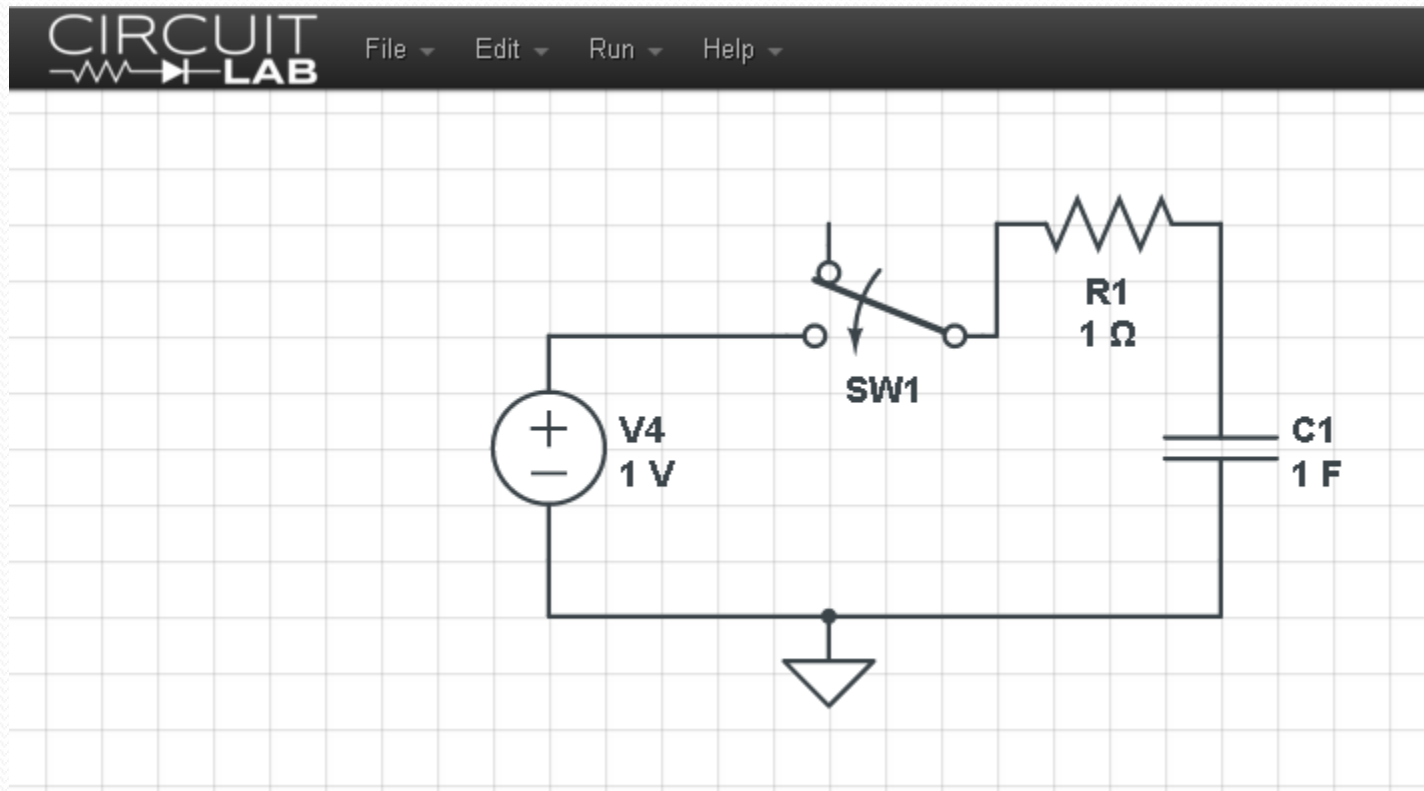


Simulation of Electrical Circs.

- Electrical technicians are able to simulate Electrical Circuits without knowledge of Differential Equations.
- However the knowledge is limited. Engineers are able to know both the electrical circuit and the differential equation that governs that circuit. The Engineering knowledge is a TOTAL knowledge of the physical phenomenon.

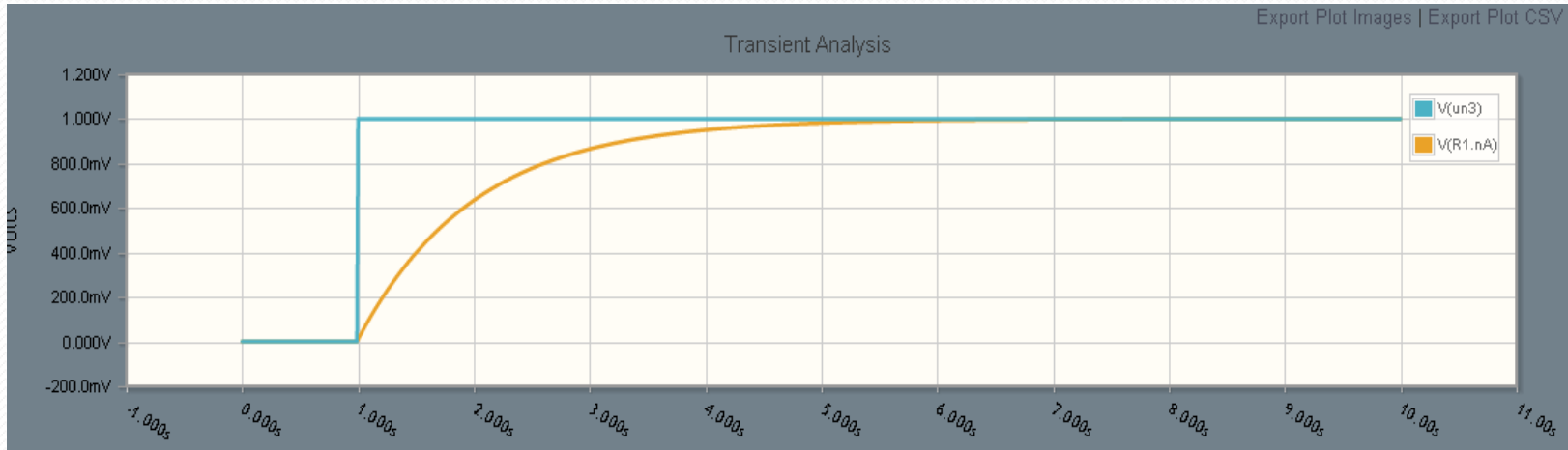
Simulation of Electrical Circs.

- To simulate Electrical circuits we use a online simulator:
 - <https://www.circuitlab.com/>
- The simulation circuit is shown in next figure!!

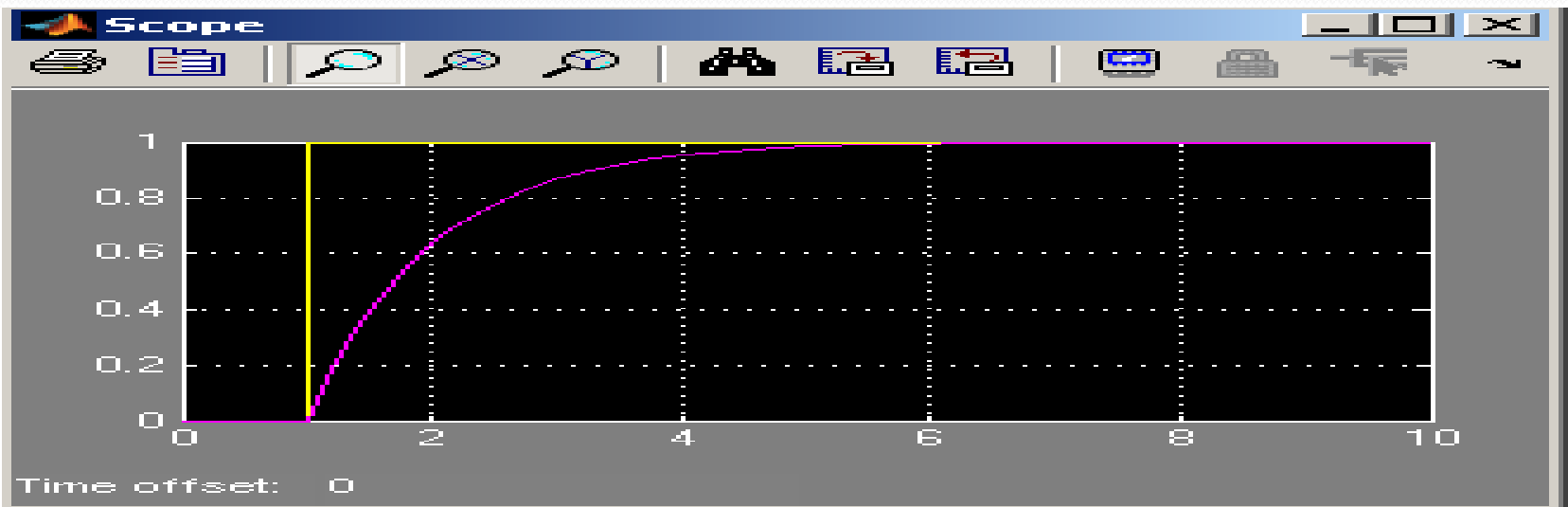


Simulation of Electrical Circs.

- Running simulation we have:



- Same as the result obtained from the Differential Eq. in Matlab





End of Class