EE 1130
Freshman Eng. Design for Electrical and Computer Eng.

Class 4

Signal Processing Module (DSP).

- Laplace Transform. Transfer Function.
- Filter Design with Zeros and Poles.

Simulink: Laplace Transform.

- Working with Differential Equations (DE) is not easy. Laplace Transform allows exchange DE for something called Transfer Function (TF). The TF gives us a direct expression of Input/Output that the DE is not able to.

$$RC\dot{y} + y = x$$

- Also, it allows us to have an direct relation input/output!!

$$RC\frac{dy}{dt} + y(t) = x(t)$$

- Aplying Laplace:

$$RCsY(s) + Y(s) = X(s)$$

Simulink: Laplace Transform.

- Operating:

$$RCsY(s)+Y(s)=X(s)$$

$$Y(s)\left(RCs+1\right)=X(s)$$

$$Y(s)=\frac{1}{RCs+1}X(s)$$

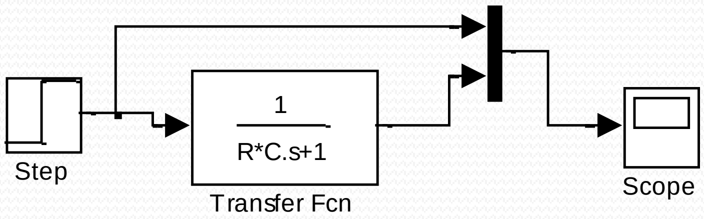
- We could easily implement this in Simulink!!!
- The multiplier of $X(s)$ is called Transfer Function.

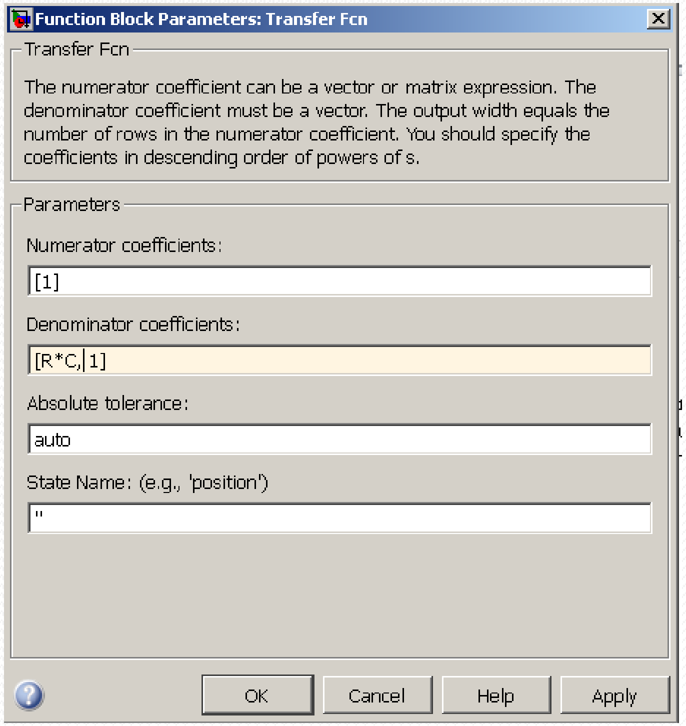
$$H(s)=\frac{1}{RCs+1}$$

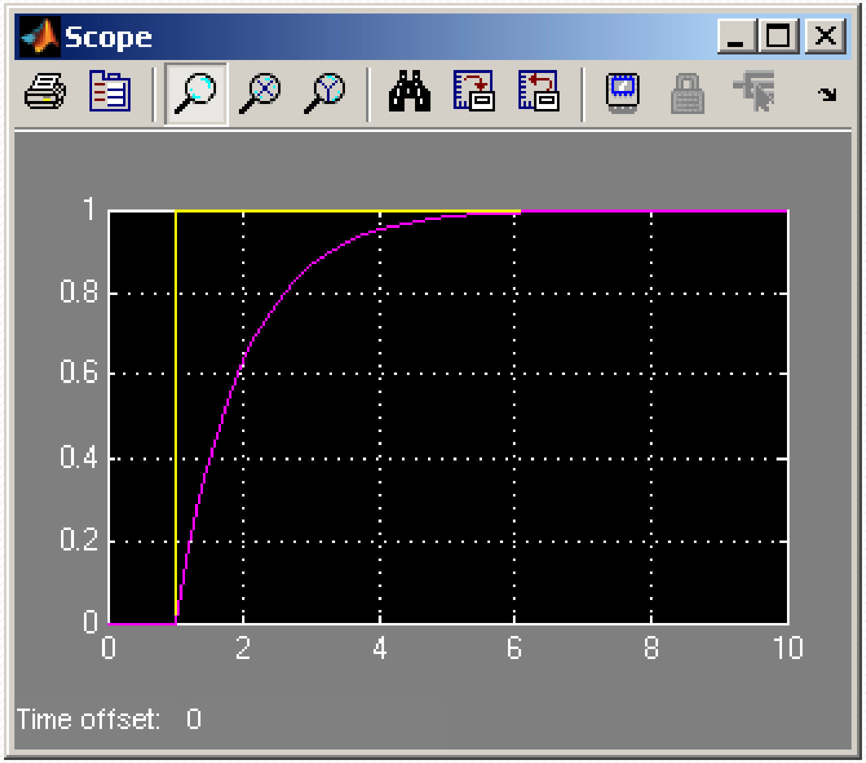
Simulink: Laplace Transform.

- Double click on Transfer Fcn to open options as shown below:
- Simulating:

$$H(s) = \frac{1}{RCs + 1}$$

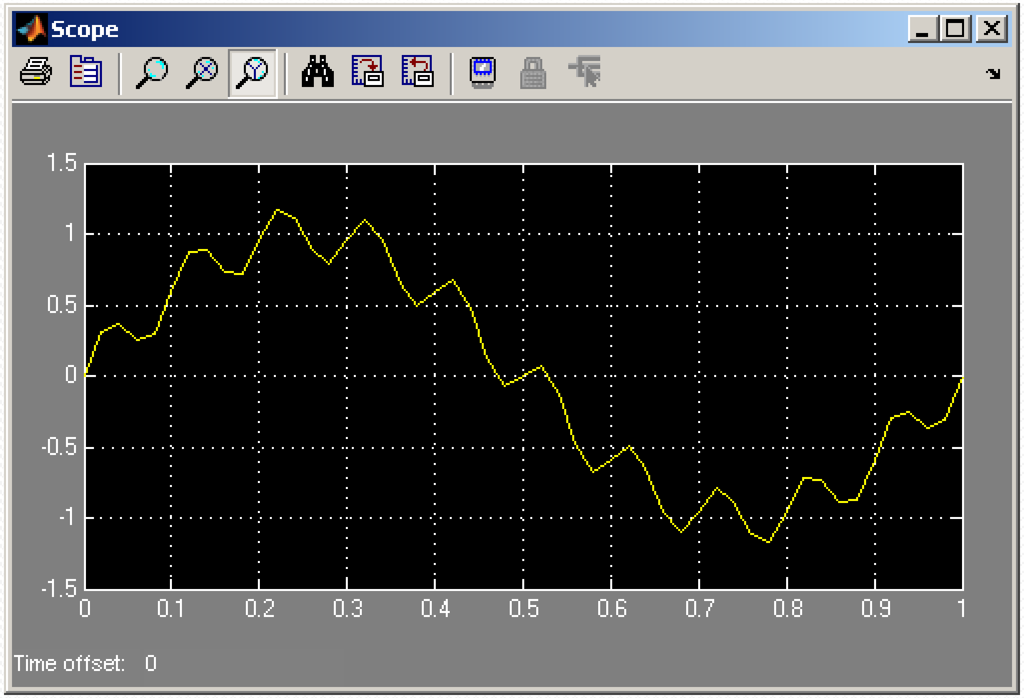






Simulink: Signal Processing.

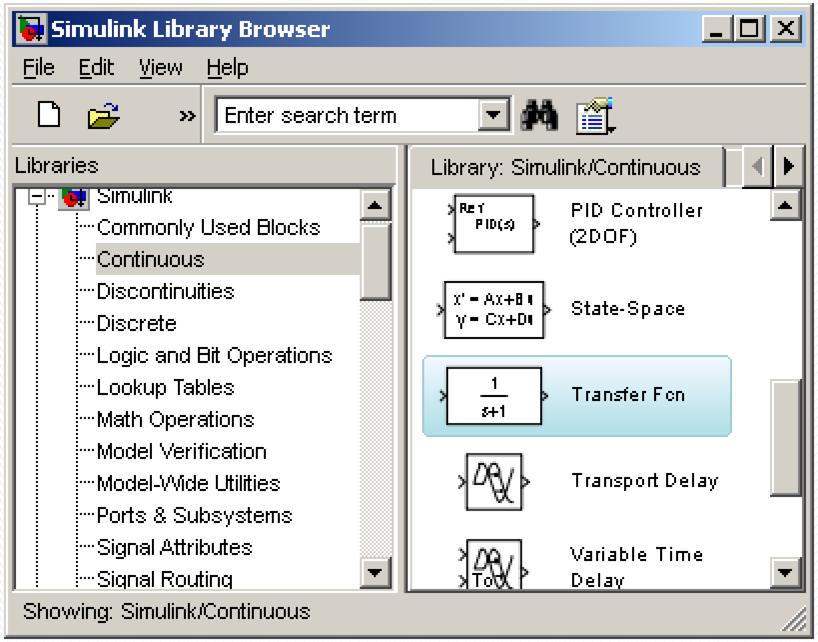
- Last lecture we ended up with a noisy signal as next figure shows:

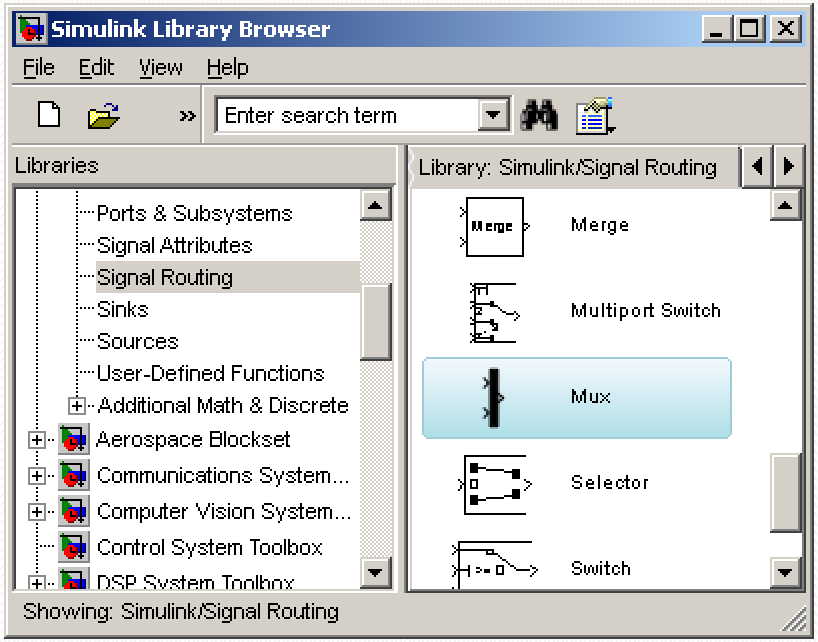


$$x(t) = \sin(2\pi 1t) + 0.2\sin(2\pi 60t)$$

Simulink: Signal Processing.

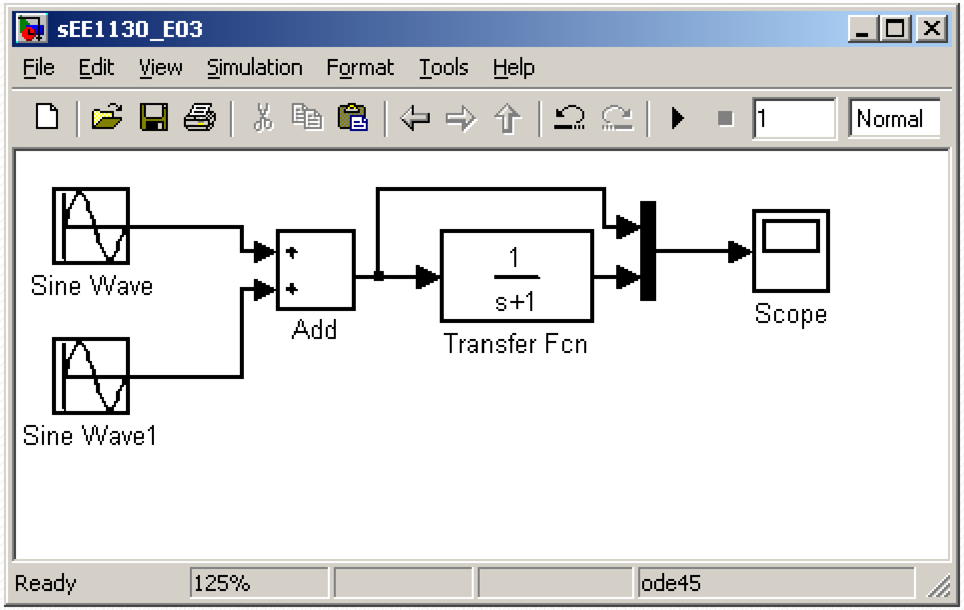
- We will insert a system that will filter out the ripple.
- First option is to insert from the continuous library group a Transfer Function block.
- We also add a Mux from Signal Routing library group.





Simulink: Signal Processing.

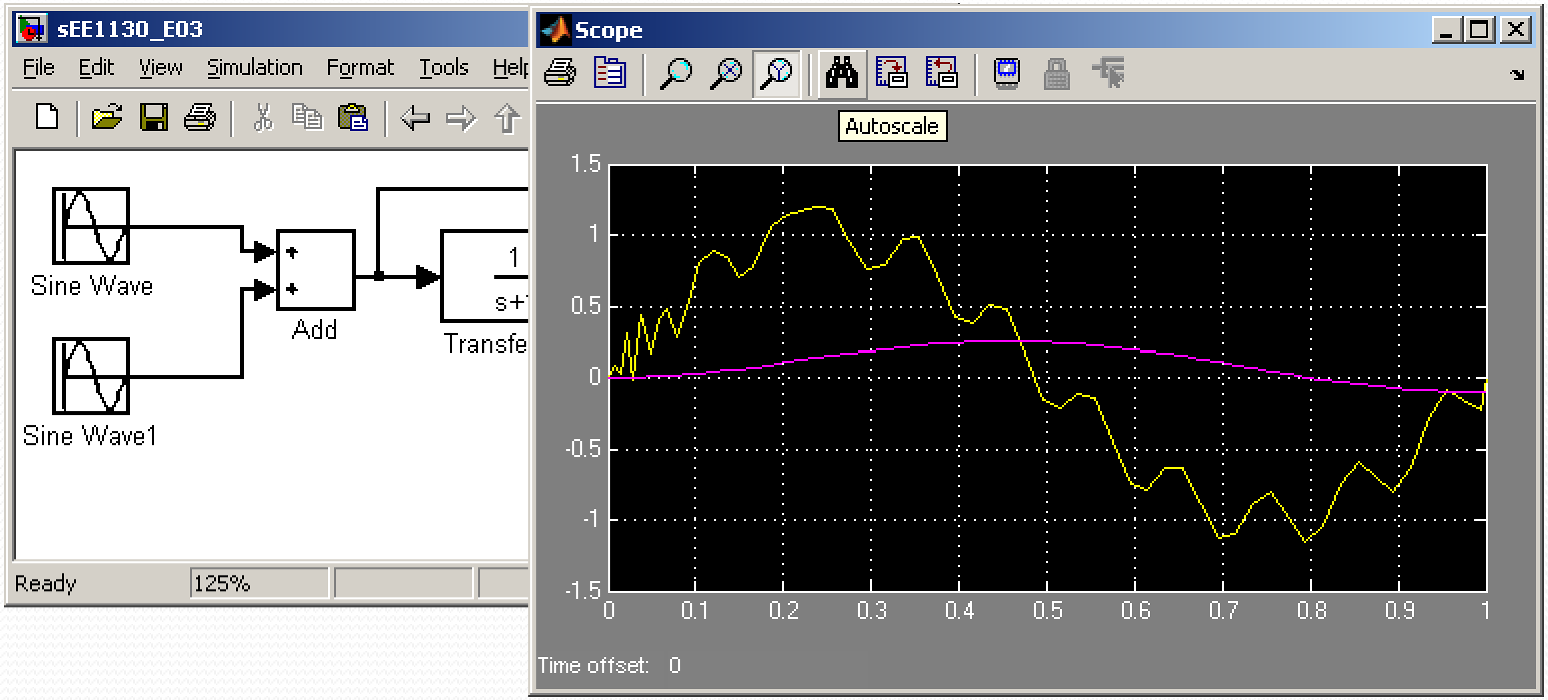
- We insert the Transfer Function after the summator and before the Mux.
- The Mux will allow the Scope to show two traces:



- Now, hit play and see:

Simulink: Signal Processing.

- Now, hit play and see:
- Somehow we cleaned the signal, but we need to amplify its gain by a factor of 4. We open the transfer function and set 4 the numerator to do this.



Simulink: Signal Processing.

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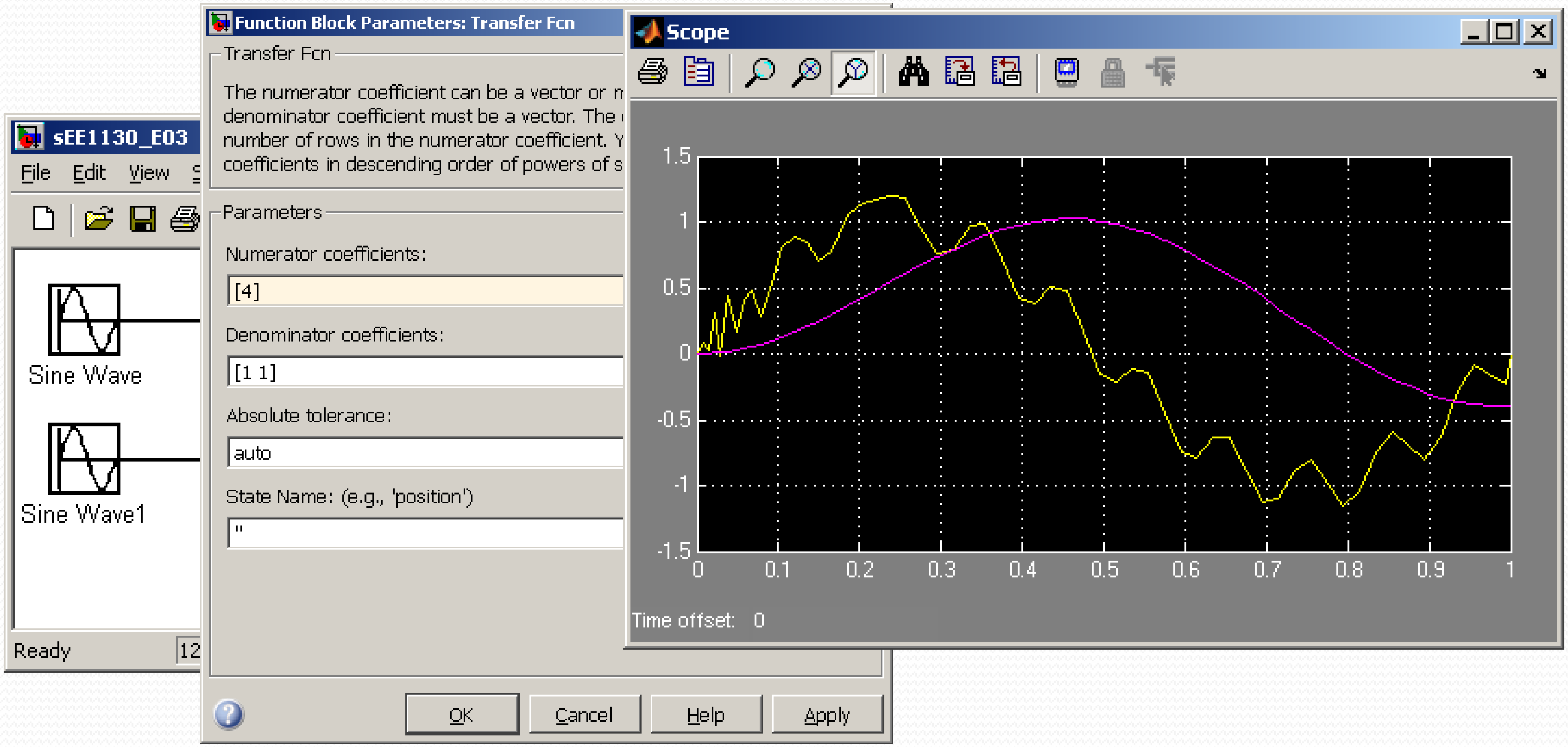
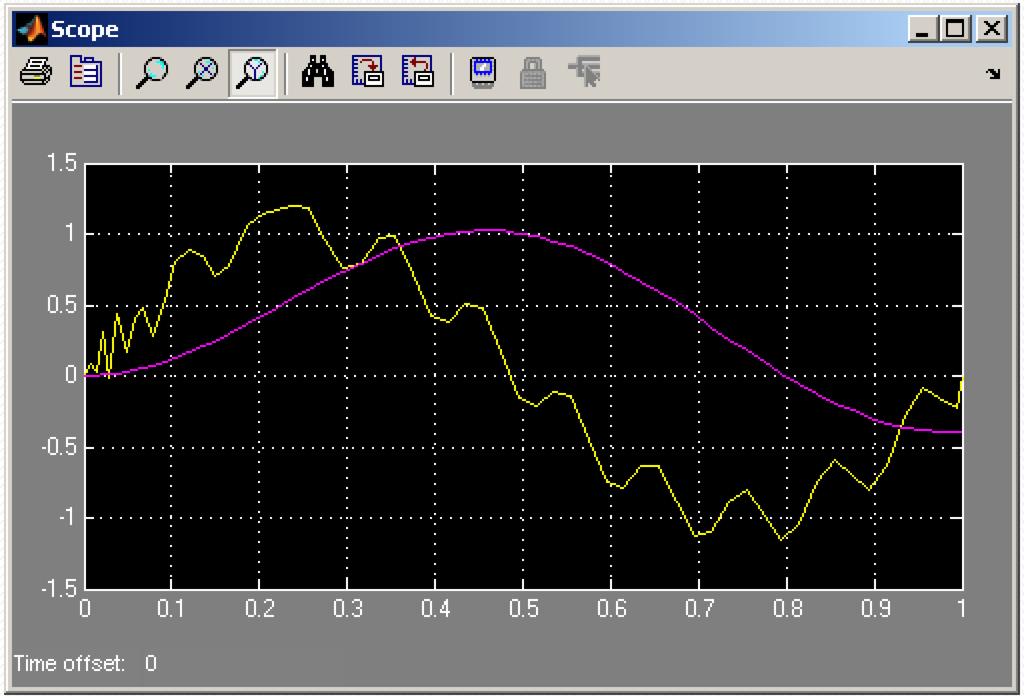


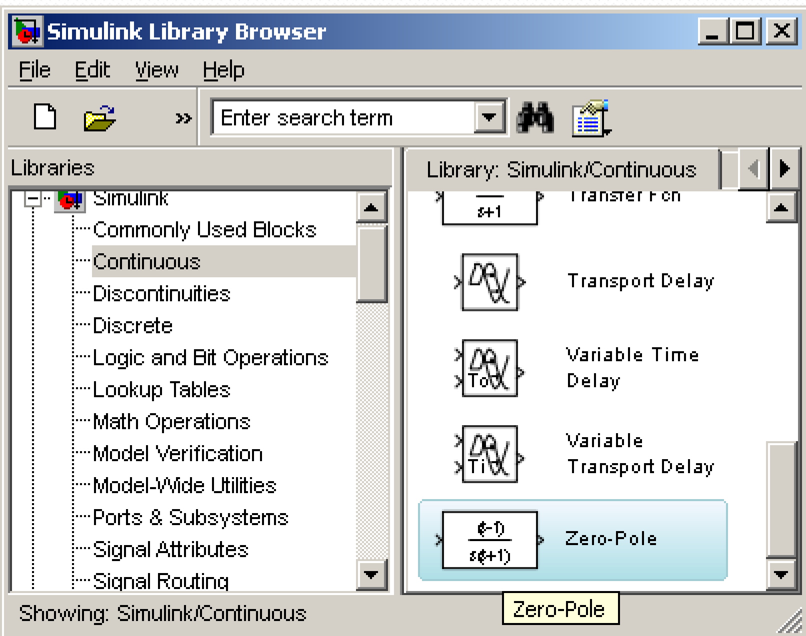
Simulink: Signal Processing.

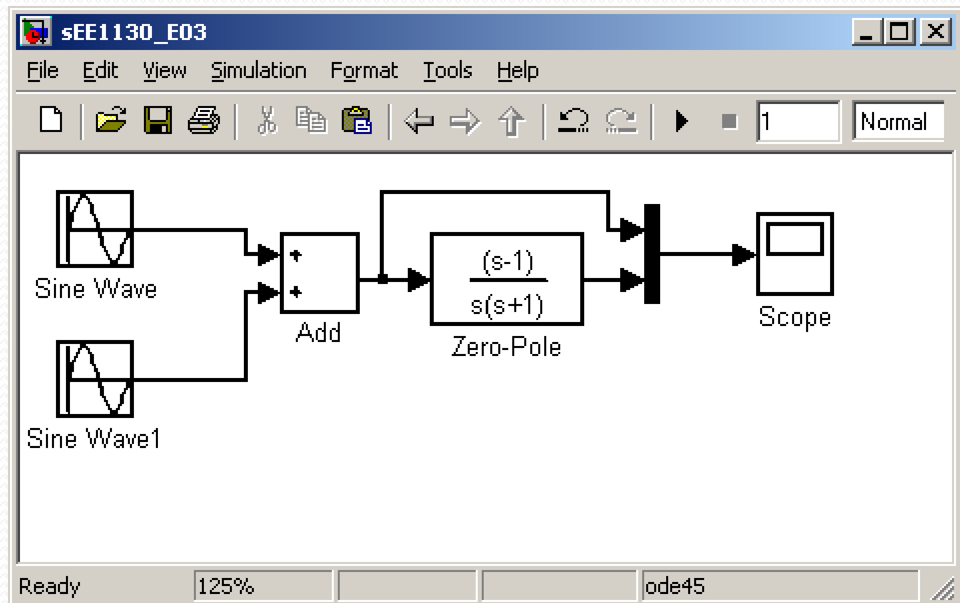
- We have cleaned the signal, but introduced a time delay (time shift). This is common in any kind of filters.



Simulink: Signal Processing.

- Now, lets design a filter that particularly eliminates the signal of 60Hz. We do that using the Zero-Pole Transfer function





Simulink: Signal Processing.

- When studying Filter Theory you will learn that the roots of the numerator (called zeros) must be $s=2\pi 60j$ where 60 is the frequency to eliminate at the output.

Simulink: Signal Processing.

- But the coefficients of the numerator are some of the values of the Electrical Components.
- Remember, for the RC circuit we had:

$$H(s) = \frac{1}{s + 1/\tau}$$

$$Y(s) = \frac{1}{s + 1/\tau} X(s)$$

$$Y(s)s + \frac{1}{RC} Y(s) = X(s)$$

$$\frac{dy}{dt} + \frac{1}{RC} y(t) = x(t)$$

Simulink: Signal Processing.

- But the coefficients of the numerator are some of the values of the Electrical Components or amplifier gains.
- However, we can not have imaginary coefficients, because they are component values or amplifier gains that MUST BE REAL.
- We need to do a mathematical trick to convert imaginary numbers into real numbers!!
  - COMPLEX CONJUGATE
  - $(a + jb)\ (a - jb) = a^2 + b^2$ eso es debido a que $-j^2 = 1$

Simulink: Signal Processing.

- When studying Filter Theory you will learn that the roots of the numerator must be (s-2π60j) and (s+2π60j). The use of complex conjugated roots is to have real coefficients because:

$$(s-2\pi 60\,j)(s+2\pi 60\,j)=s^2+4\pi^2 60^2$$

- At the denominator we just set roots to.

$$(s+340)(s+360)$$

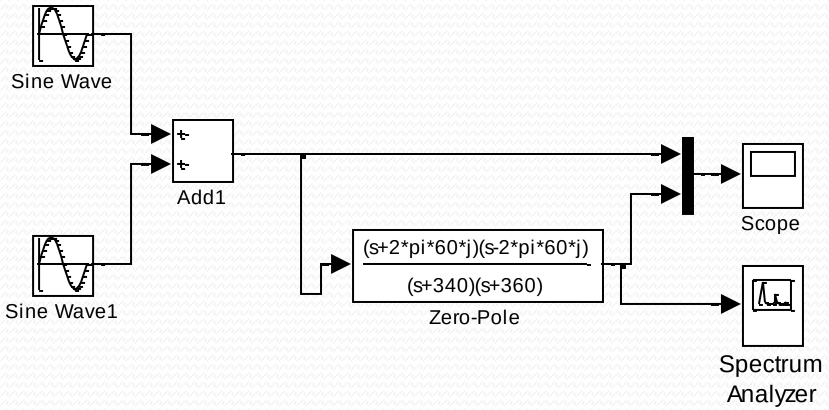
- If you set smaller roots, the output becomes too large. Please try other values to check out by yourself

Simulink: Signal Processing.

- The final Transfer Function that solve our problem is:

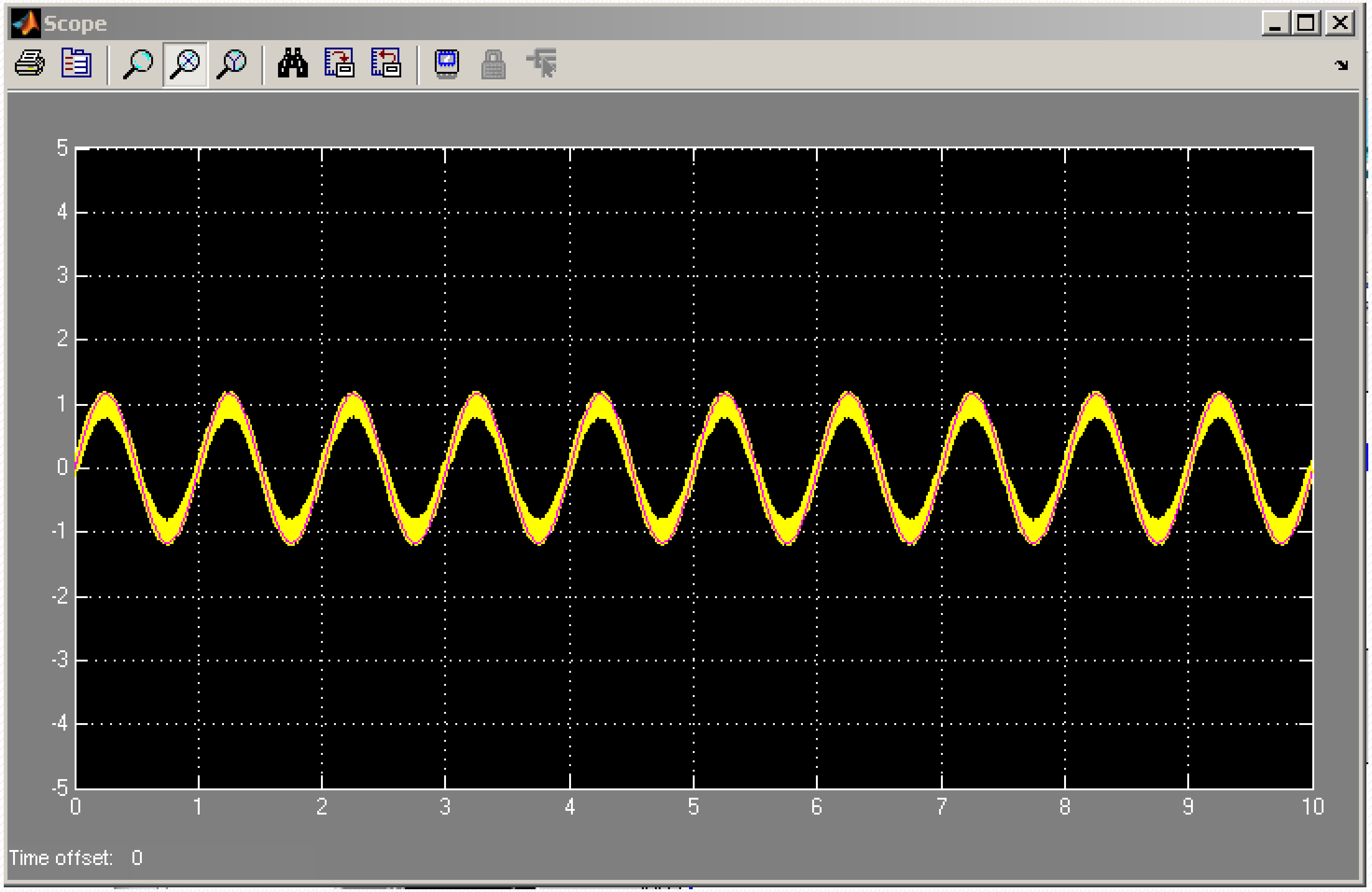
$$H(s) = \frac{s^2 + 4\pi^2 60^2}{(s+340)(s+360)}$$

- Now, we simulate this in Simulink



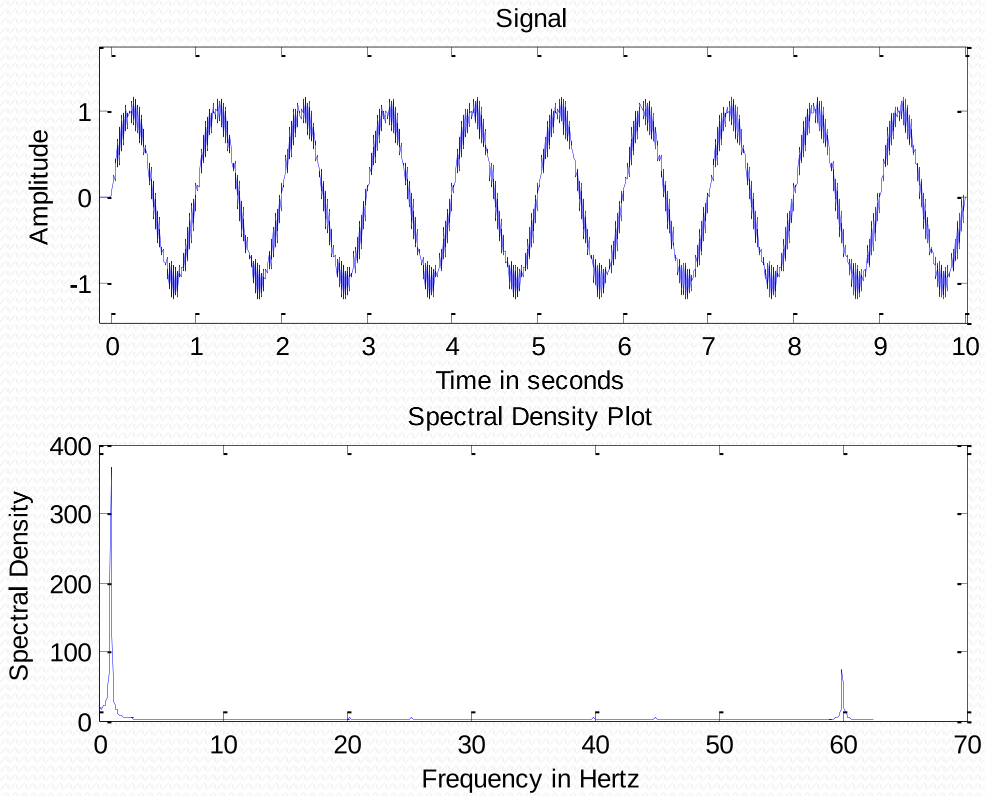
Simulink: Signal Processing.

- Now we hit play and compare input and output in the Scope



Simulink: Signal Processing.

- The simulation shows we did the job
  - Spectrum before the filter



Simulink: Signal Processing.

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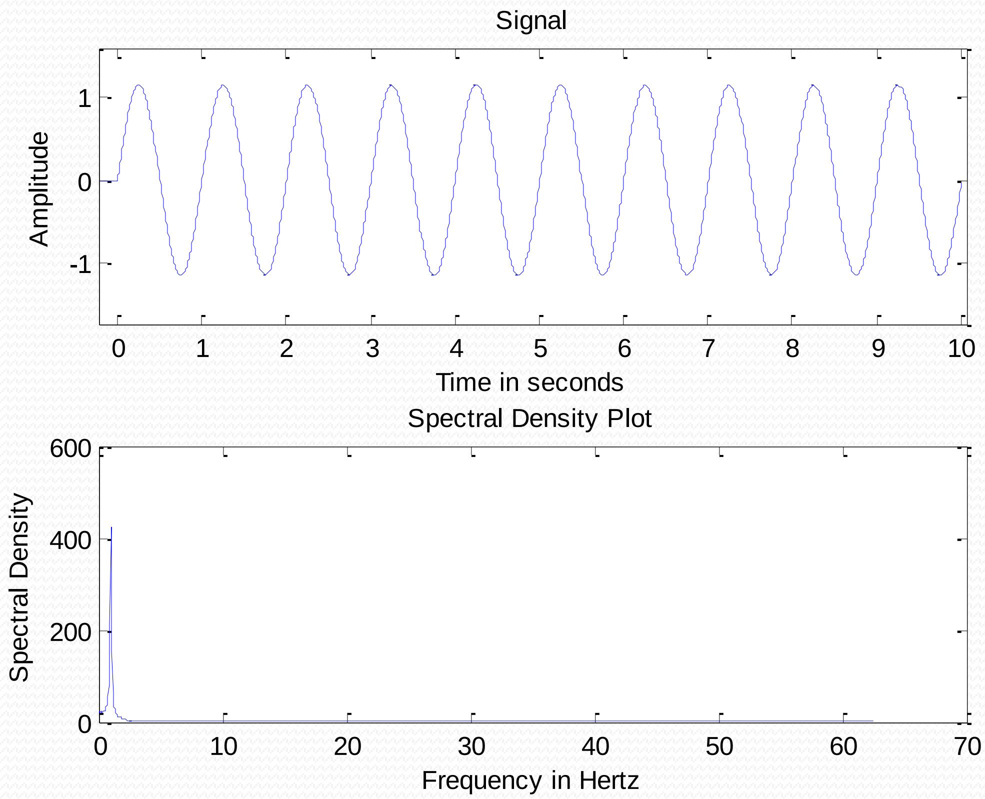
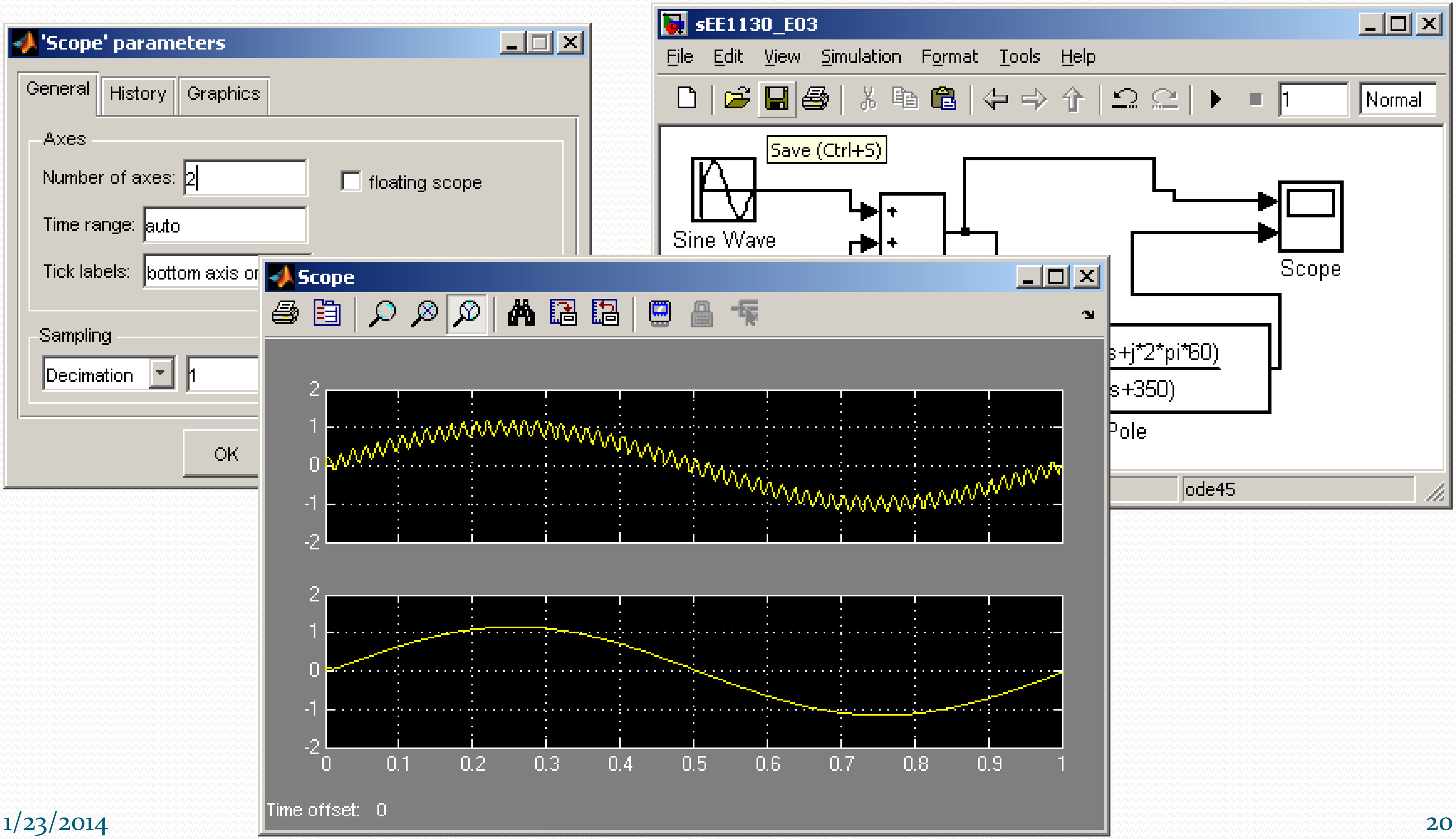


Simulink: Signal Processing.

- We notice the dark trace is completely clean of noise. We could add another trace to the scope and see both signals separated:



Simulink: Signal Processing.

- Once the simulation shows we solved the problem, we need to implement the Electrical Circuit.
- In order to do that, we need to modify the Transfer Function in a sum of simpler Transfer Functions of the type:

$$H_{simple}(s) = \frac{G}{(\tau s + 1)}$$

- This is done with Partial Fraction Expansion:

$$H(s) = \frac{s^2 + 4\pi^2 60^2}{(s+340)(s+360)} = \frac{R_1}{s+340} + \frac{R_2}{s+360}$$

- Matlab calculate the residues very fast:

Simulink: Signal Processing.

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$$H(s) = \frac{s^2 + 4\pi^2 60^2}{(s+340)(s+360)} = \frac{-1.35861*10^4}{s+360} + \frac{1.2886*10^4}{s+340}$$

Simulink: Signal Processing.

- One more modification yields:

$$H(s) = \frac{-37.7}{\frac{1}{360}s+1} + \frac{37.9}{\frac{1}{340}s+1}$$

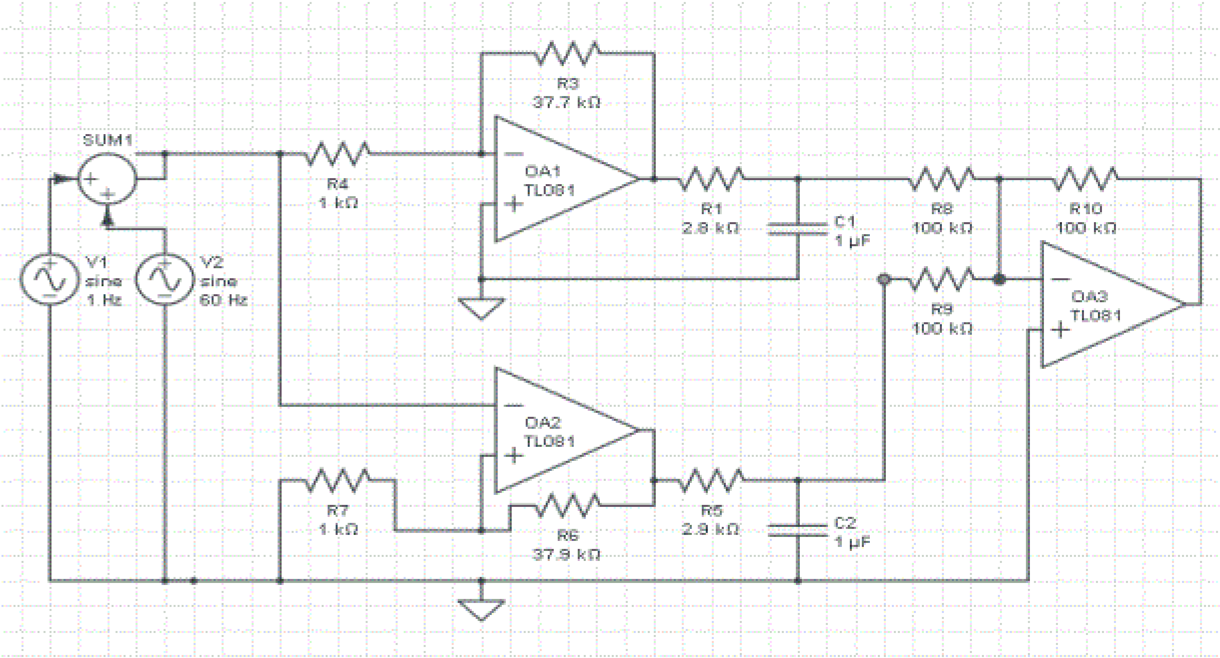
- Each term correspond to a RC circuit:

$$H_{simple}(s) = \frac{G_1}{(R_1C_1s+1)} + \frac{G_2}{(R_2C_2s+1)}$$

Simulink: Signal Processing.

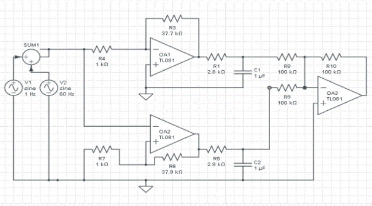
- Implementation:

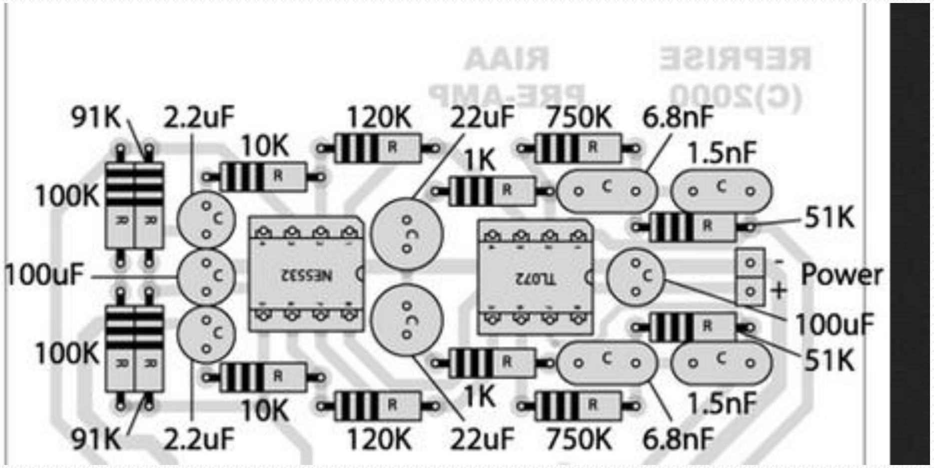
$$H(s) = \frac{-37.7}{\frac{1}{360}s+1} + \frac{37.9}{\frac{1}{340}s+1}$$



Simulink: Signal Processing.

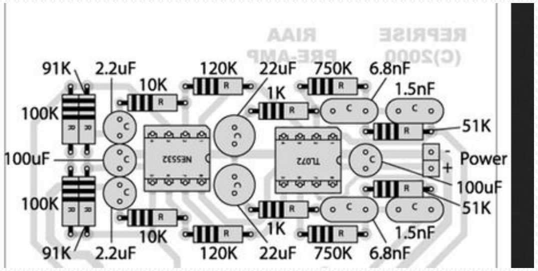
- From the Electrical Schematics we build the physical layout:
- We obtain something like:





Simulink: Signal Processing.

- From the physical layout:
- we build the PCB (Printed Circuit Board)



- We solder the components.
- Solder the cables.

- Then we test!!!

Simulink: Signal Processing.

- We generate the final report with our findings, to assess that the circuit does what we intended it to do.

End of Class