EE 1130

Freshman Eng. Design for Electrical and Computer Eng.

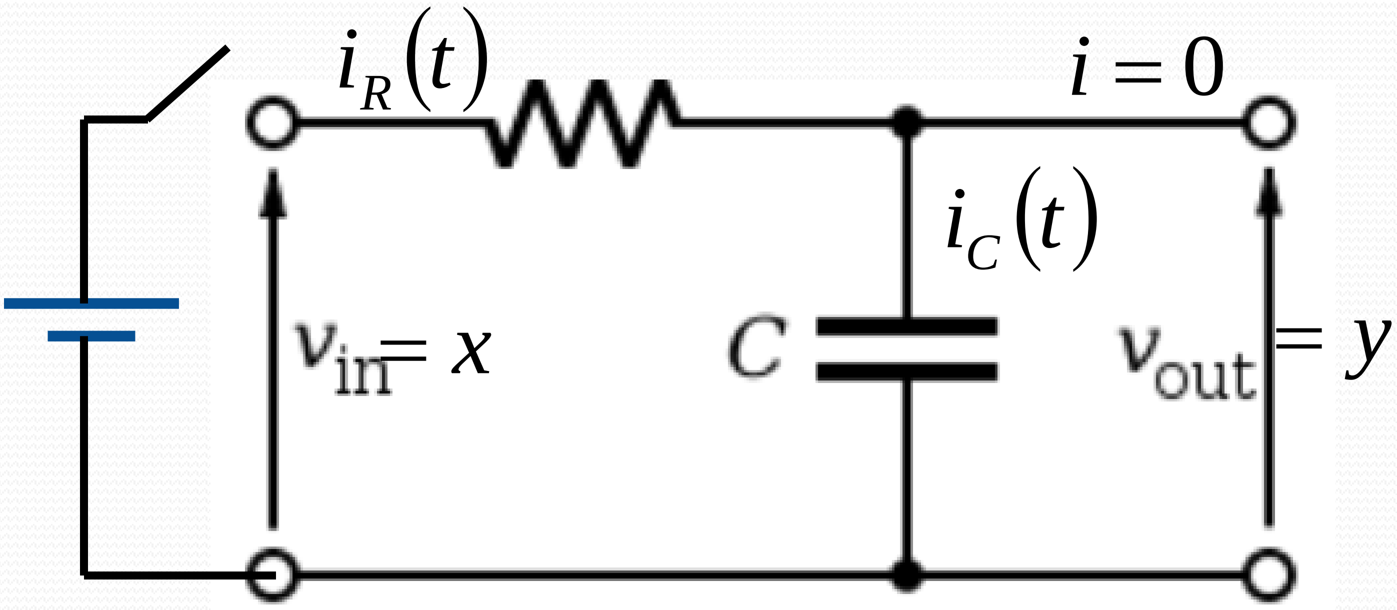
Class 3

Signal Processing Module (DSP).

- Differential Equations.
- Laplace Transform. Transfer Function.
- Simulink with Transfer Functions. Zeros, Poles.

Simulink: Differential Equations.

- Any Linear Time Invariant system could be modeled as the solution of a differential equation (DE) .
- In the case of Low Pass RC filter shown in next figure:



- The Differential Equation is:

$$RC\dot{y} + y = x$$

Simulink: Differential Equations.

- A differential equation is not instant.

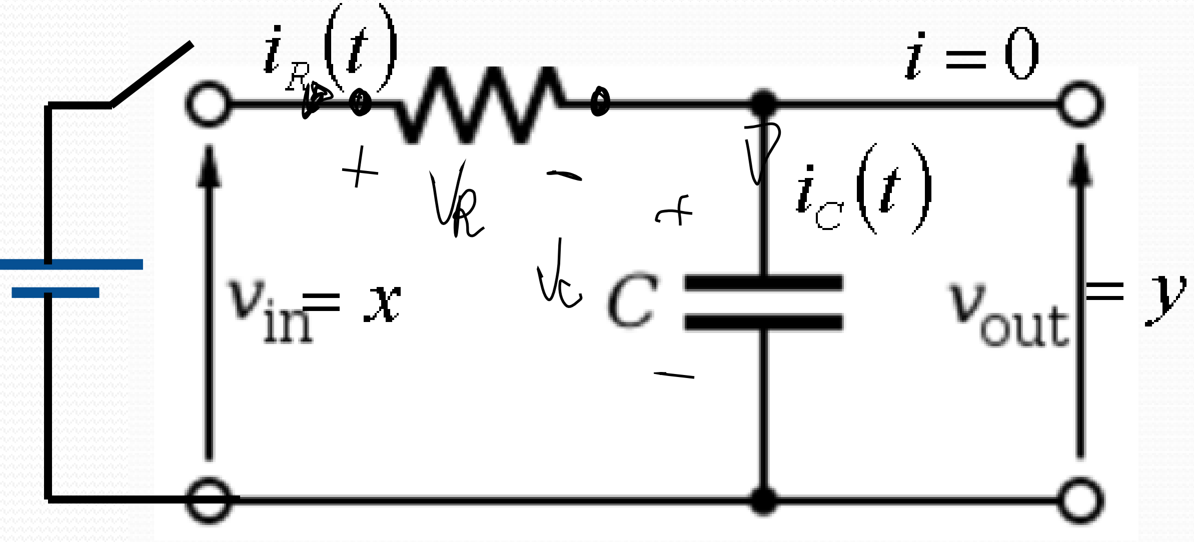
$$y(t) = 2x(t)$$

- A differential equation has into account velocities!!!

$$K\frac{dy}{dt} + y(t) = 2x(t)$$

Simulink: Differential Equations.

- The circuit analysis is shown in next figure:

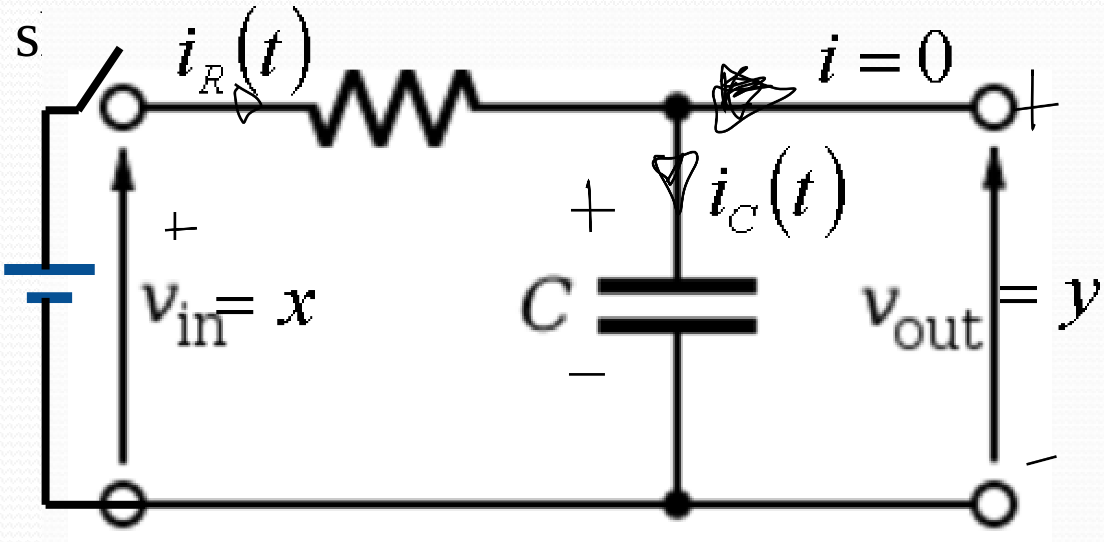


$$y(t) = v_C(t)$$

$$i_R(t) = \frac{v_R(t)}{R} \qquad i_C(t) = C\frac{dv_C}{dt}$$

Simulink: Differential Equations.

- The circuit analysis is s



$$x(t) = v_R(t) + v_C(t)$$

$$x(t) = i_R(t)R + y(t)$$

$$x(t) = \left(C\frac{dy}{dt} \right) R + y(t)$$

$$RC\frac{dy}{dt} + y(t) = x(t)$$

Simulink: Differential Equations.

$$RC\dot{y} + y = x$$

- Where y with the dot is the first derivative of $y(t)$ and x is $x(t)$. R and C are the values of the Resistor and Capacitor respectively.
- The Differential Equation could be simulated with Simulink.
- However, the Differential Equation must be modified to an Integral Equation, since integrator blocks are more used than derivative blocks.

$$\int \left(RC\dot{y} + y\right) dt = \int x\, dt$$

Simulink: Differential Equations.

- The integral is linear:

$$RC\int \dot{y}\,dt + \int y\,dt = \int x\,dt$$

$$RCy = \int x\,dt - \int y\,dt$$

$$y = \frac{1}{RC}\int (x - y)dt$$

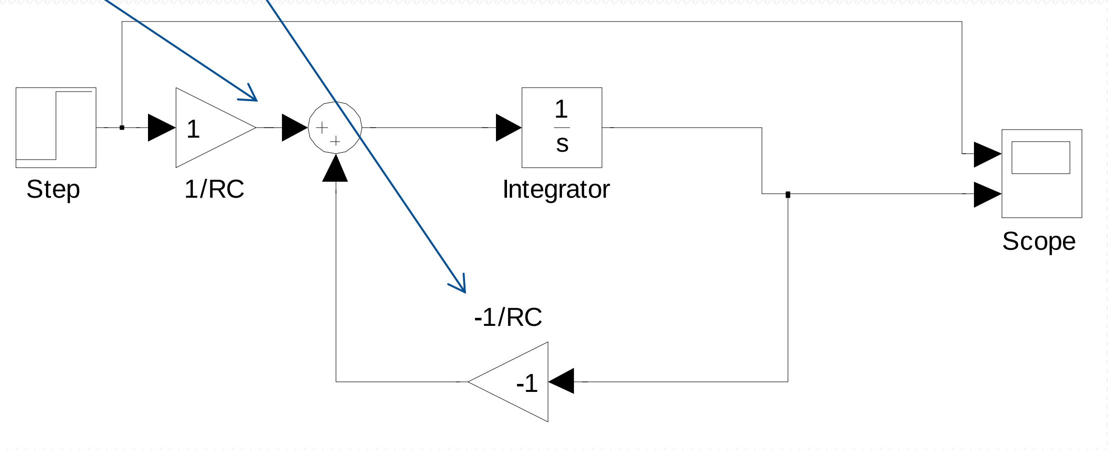
$$y = \int \left(\frac{1}{RC}x - \frac{1}{RC}y\right)dt$$

Simulink: Differential Equations.

- The block diagram could be implemented from this equation:

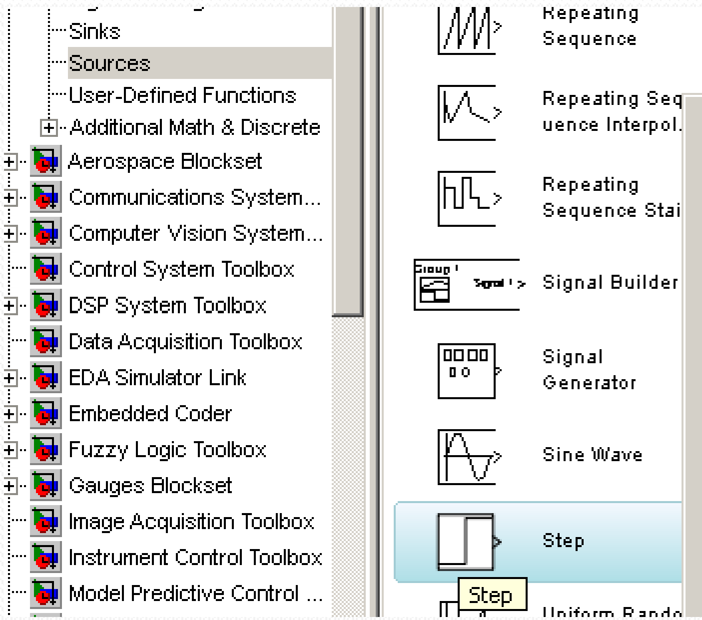
$$y = \int \left(\frac{1}{RC} x - \frac{1}{RC} y \right) dt$$

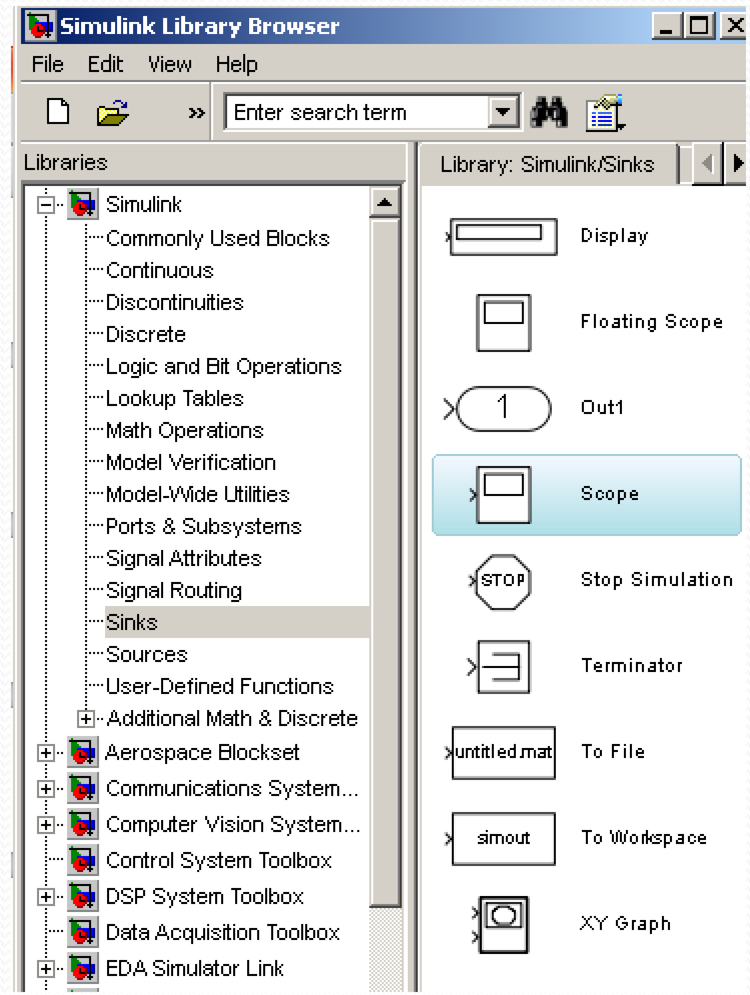
$$\frac{1}{RC} x - \frac{1}{RC} y$$



Simulink: Differential Equations.

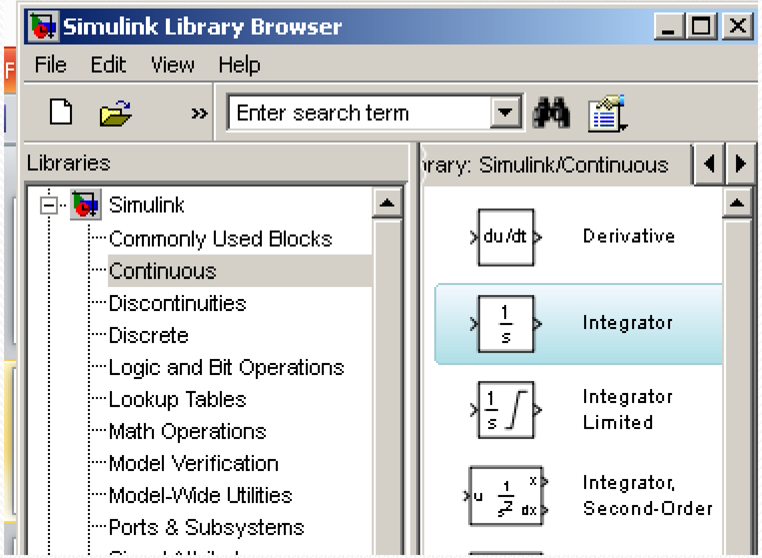
- To insert the Step and Scope blocks we do:





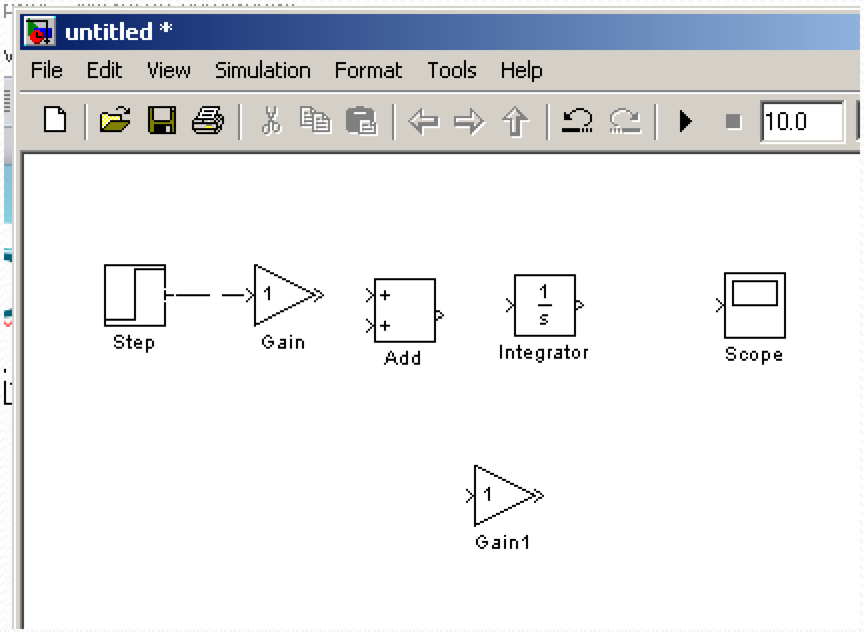
Simulink: Differential Equations.

- To insert the Integrator block we do:

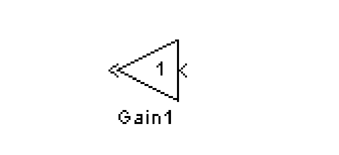


Simulink: Differential Equations.

- Once all elements in the Model, we make the connections:



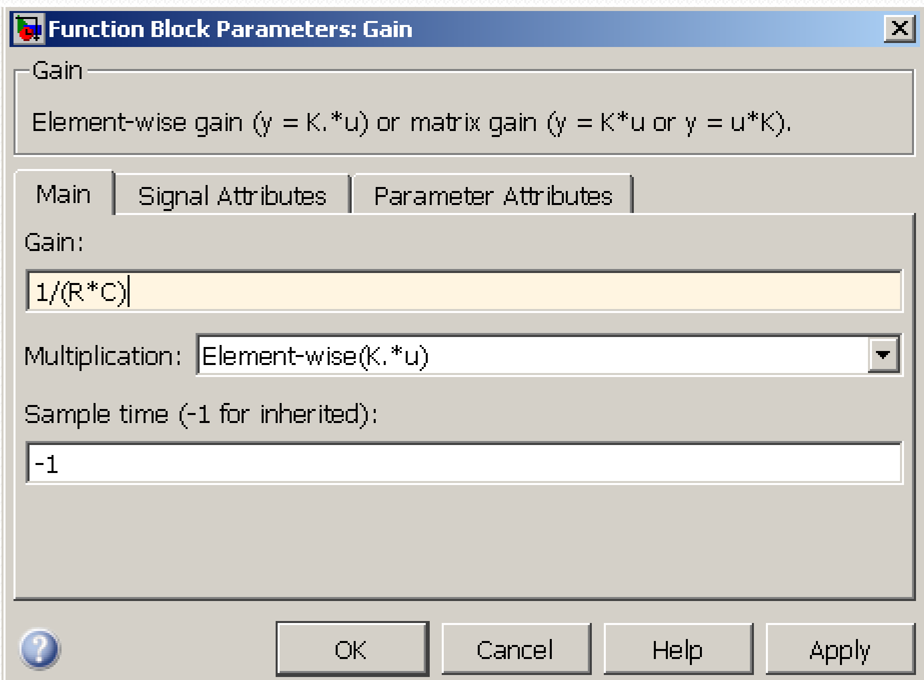
- To flip the Gain1 block we type control+I

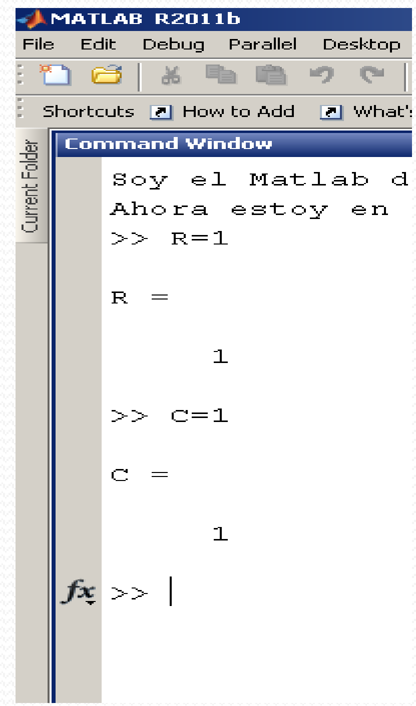


Simulink: Differential Equations.

- Double click on each gain block and change the 1 to 1/(R*C) at the first block and -1/(R*C) at the second.

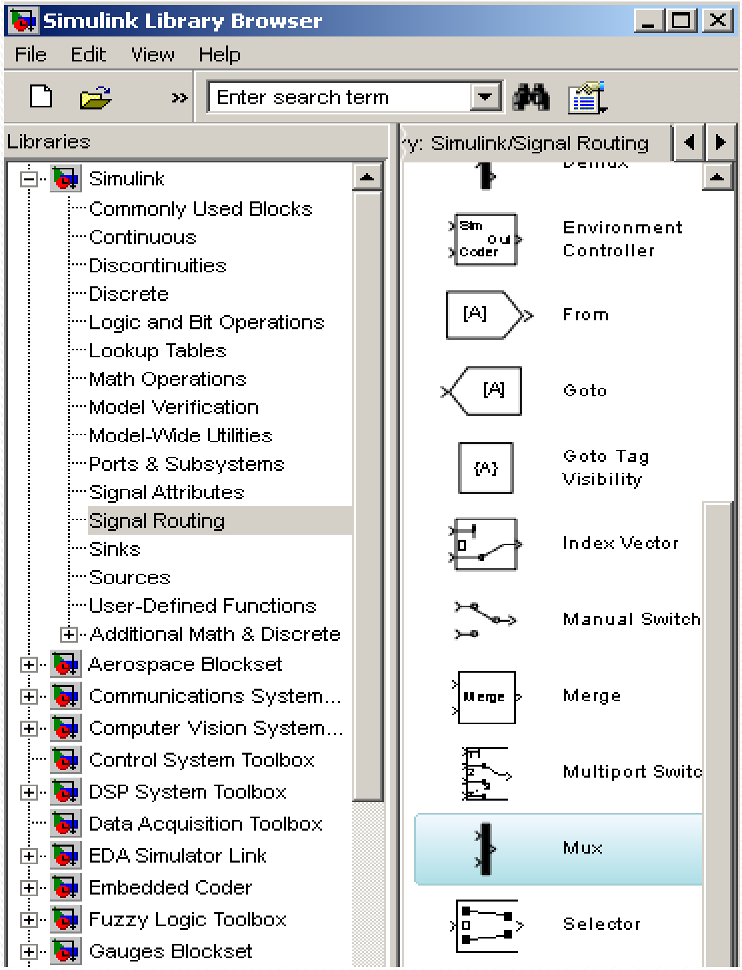
- Type C=1 and R=1 at the command window to define the variables R and C.





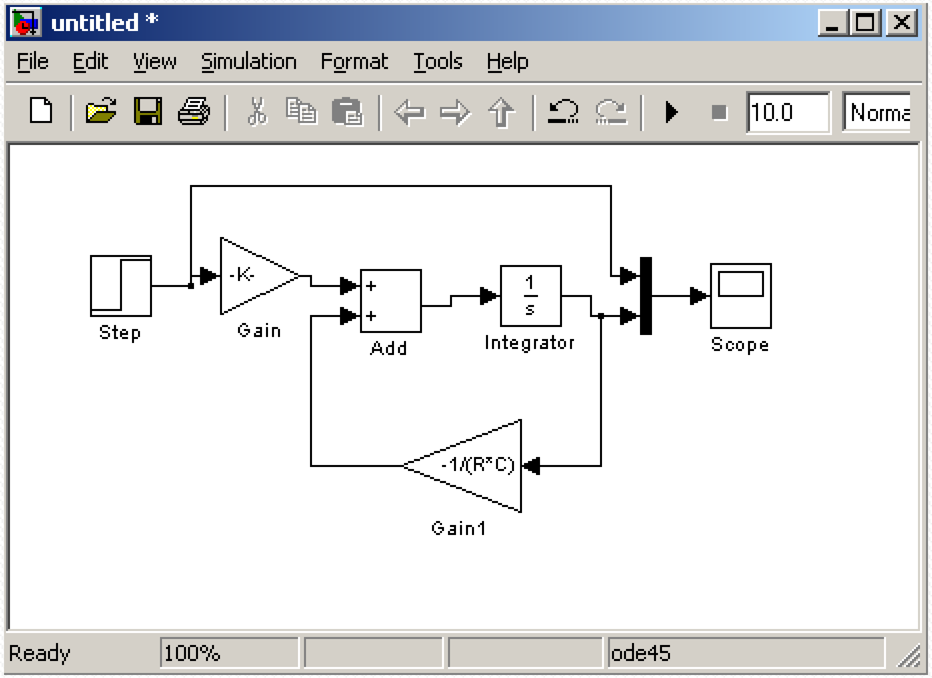
Simulink: Differential Equations.

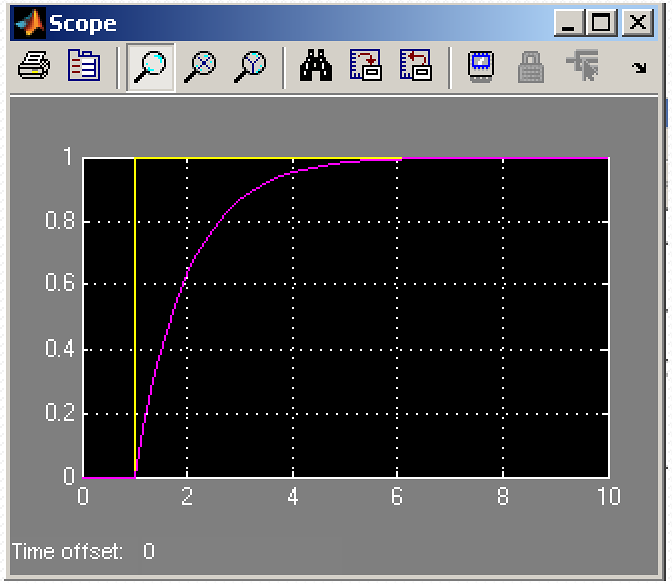
- To see both traces in one scope we add the MUX.



Simulink: Differential Equations.

- Once all connected, variables defined in command window, we hit play and double click on the scope block to open the scope screen.





Simulink: Laplace Transform.

- Working with DE is not easy. Laplace Transform allows avoid DE.
- Also, it allows us to have an analytic relation input/output!!

$$RC\dot{y} + y = x$$

$$RC\frac{dy}{dt} + y(t) = x(t)$$

- Aplying Laplace:

$$RCsY(s) + Y(s) = X(s)$$

Simulink: Laplace Transform.

- Operating:

$$Y(s)\left(RCs+1\right)=X(s)$$

$$Y(s)=\frac{1}{RCs+1}X(s)$$

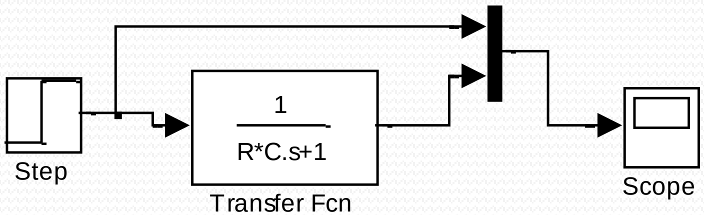
- We could easily implement this in Simulink!!!
- The multiplier of $X(s)$ is called Transfer Function.

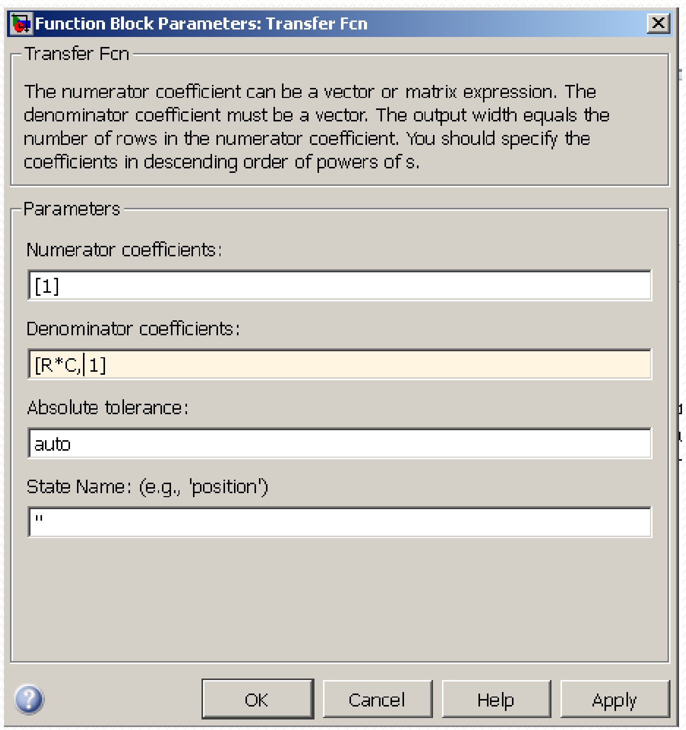
$$H(s)=\frac{1}{RCs+1}$$

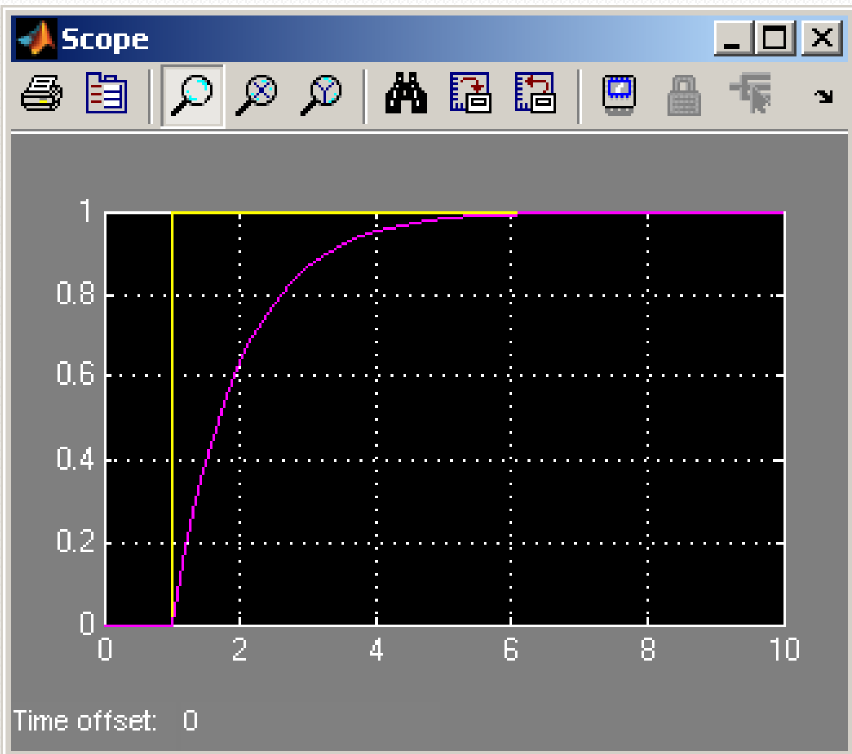
Simulink: Laplace Transform.

- Double click on Transfer Fcn to open options as shown below:
- Simulating:

$$H(s)=\frac{1}{RCs+1}$$







End of Class